What's Going On?

Checking In	
Minds on	Expand
Action!	Expand Again
Consolidation	Expand Some More

Learning Goal - I will be able to use Pascal's Triangle to expand binomials.

Checking In

LGL

The third term of a geometric sequence is 32,
the seventh term is 8,192.

What is the sum of the first 10 terms?

S_n

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \text{or} \quad S_n = \frac{t_{n+1} - t_1}{r-1}$$

1. Find r

$$32 \times r^4 = 8192$$

$$r^4 = \frac{8192}{32}$$

$$\sqrt[4]{r^4} = \sqrt[4]{256}$$

$$r = 4$$

2. Find a

$$a \times 4^2 = 32$$

$$a = 2$$

$$S_n = \frac{2(4^{10} - 1)}{4 - 1}$$

$$= \frac{2(1048575)}{3}$$

$$= 699050$$

Minds on

Expand

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x + 1y$$

$$(x+y)^2 = 1x^2 + 2xy + 1y^2$$

Action!

Expand Again

Expand, and arrange terms in descending degree of x

$$(x+y)^3 =$$

$$(x+y)^4$$

$$(x+y)^5$$

$(x+y)^1$ $(x+y)^2$

$$(x+y)^3$$

$$= (x+y)(x^2+2xy+y^2)$$

$$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$$

$$= 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

$$(x+y)^4$$

$$= (x+y)(1x^3 + 3x^2y + 3xy^2 + 1y^3)$$

$$= \cancel{1x^4} + \cancel{3x^3y} + \cancel{3x^2y^2} + 1xy^3 + \cancel{1x^3y} + \cancel{3x^2y^2} + 3xy^3 + 1y^4$$

$$= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

Action!

Pascal's Triangle

Write out the coefficients of each term in the expansions given below. Be sure to arrange your terms in descending order of x.

$$(x+y)^0$$

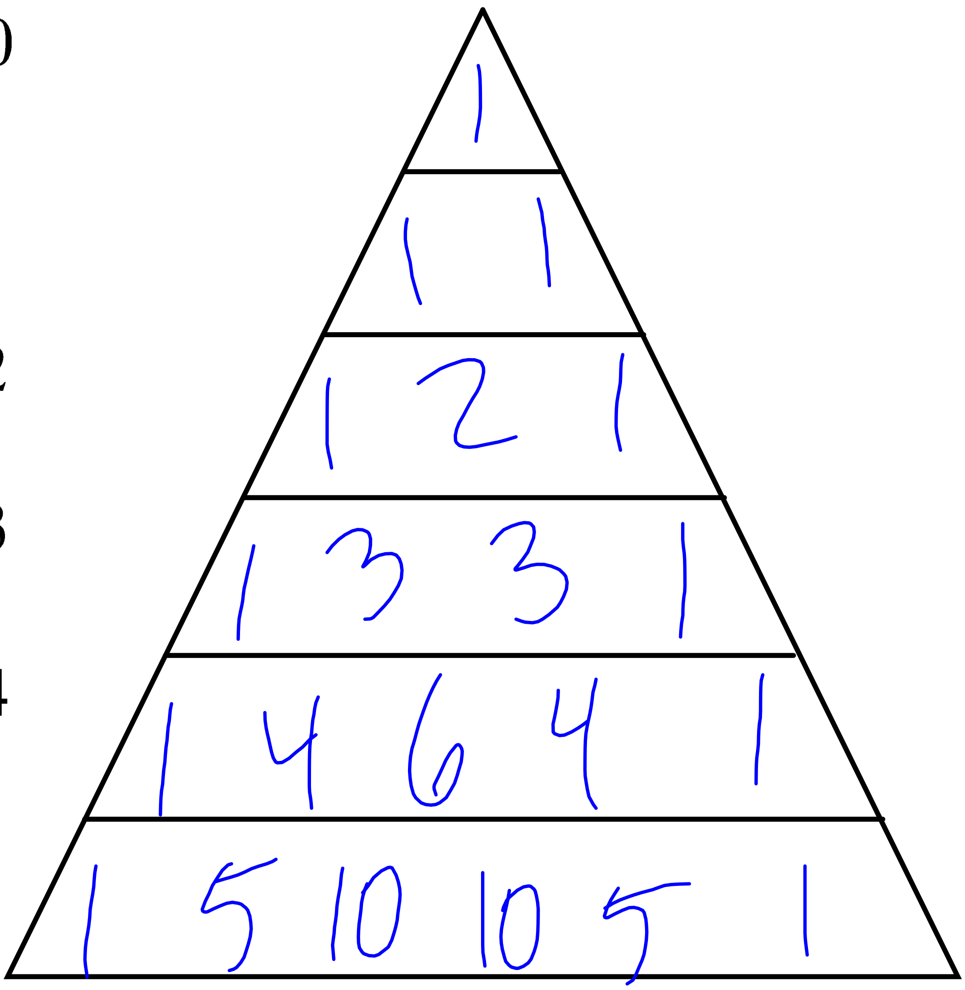
$$(x+y)^1$$

$$(x+y)^2$$

$$(x+y)^3$$

$$(x+y)^4$$

$$(x+y)^5$$



Action!

Expand Again

Expand, without actually expanding, and arrange terms in descending degree of x.

$$(x+y)^6$$

Action!

Pascal's Triangle

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

Action!

Pascal's Triangle

Patterns in Pascal's Triangle

There are patterns in Pascal's Triangle and in the expansion of $(a + b)^n$

1. Each term in the expansion of $(a + b)^n$ is the product of a number from Pascal's Triangle, a power of *a,* and a power of *b*.
2. The coefficients on the terms correspond to the numbers in the n^{th-} row in Pascal's Triangle.
3. In the expansion, the exponents of *a* start at *n* and decrease to 0. The exponents of *b* start at 0 and increase to *n*.
4. The exponents on *a* and *b* always add to *n*.

Consolidation

Using the Triangle

Expand

$$(x-2)^5$$

$a = x$

$b = -2$

$= 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

$= 1(x)^5 + 5(x)^4(-2) + 10(x)^3(-2)^2 + 10(x)^2(-2)^3 + 5(x)(-2)^4 + 1(-2)^5$

$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Consolidation

Using the Triangle

Expand

$$(5x+2y)^3$$

$\boxed{a=5x}$

$\boxed{b=2y}$

$= 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$= 1(5x)^3 + 3(5x)^2(2y) + 3(5x)(2y)^2 + 1(2y)^3$

$= 125x^3 + (3\times25\times2)x^2y + (3\times5\times4)xy^2 + 8y^3$

$= 125x^3 + 150x^2y + 60xy^2 + 8y^3$

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1