What's Going On?

Checking In

Minds on   Discovering the Formula

Action!   Finding Sums

Consolidation   Unit Formulae

**Learning Goal - I will be able to calculate the sum of
a geometric series.**

LGL

Determine the sum of the arithmetic series given below.

6 + 11 + 16 + ... + 441 + 446 + 451

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$t_n = a + (n-1)d$$

$$451 = 6 + (n-1)(5)$$
$$-6 \qquad -6$$

$$\frac{445}{5} = \frac{(n-1)(5)}{5}$$

$$89 = n - 1$$
$$+1 \qquad +1$$

$$n = 90$$

$$S_n = \frac{90(6 + 451)}{2}$$

$$S_n = 45(457)$$

$$S_n = 20{,}565$$

Minds on

Sum of a Geometric Series

Write out the first *n* terms of a geometric series.

$$1 \quad 2 \quad 3 \quad 4 \qquad\qquad t_{n-1} \quad t_n$$

$$S_n = \quad a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1}$$

Minds on

Sum of a Geometric Series

Multiply each term in your series by r.

$$rS_n = \quad ar + ar^2 + ar^3 + ... + ar^{n-2} + ar^{n-1} + ar^n$$

$$-S_n = \quad a + ar + ar^2 + ar^3 + ... + ar^{n-2} + ar^{n-1}$$

$$rS_n - S_n = -a + ar^n$$

common factor

$$S_n(r - 1) = a(-1 + r^n)$$

$$\frac{S_n(r - 1)}{(r - 1)} = \frac{a(r^n - 1)}{(r - 1)}$$

$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$

Sum of a Geometric Series

Subtract the original series from your new series.

$$rS_n = \qquad ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$$

$$- \; S_n = \quad a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1}$$

Action!

The Formulae

Geometric

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$S_n = \frac{t_{n+1} - t_1}{r - 1}$$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$S_n = \frac{ar^n - a}{r - 1}$$

$$S_n = \frac{t_{n+1} - t_1}{r - 1}$$

the $n+1^{th}$
term

first
term

Action!

Using the Formulae

At a fish hatchery the number of fish that hatched on each of the first four days after fertilization was 2, 10, 50 and 250. How many fish will hatch in the first 10 days?

$$r = \frac{10}{2}$$

$$r = 5$$

$$n = 10 \qquad a = 2$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{(2)(5^{10} - 1)}{5 - 1}$$

$$S_n = \frac{(2)(9765624)}{4}$$

$$S_n = 4{,}882{,}812$$

| **Action!**

Using the Formulae

Calculate the sum of the geometric series

7,971,615 + 5,314,410 + 3,542,940 + ... + 92,160

$a = 7971615$ $r = \dfrac{3542940}{5314410}$

$r = 0.666666$

$r = \dfrac{2}{3}$

$t_{n+1} = 92160 \times \dfrac{2}{3}$

$= 61440$

$S_n = \dfrac{61440 - 7971615}{\frac{2}{3} - 1}$

$S_n = \dfrac{-7910175}{-\frac{1}{3}}$

$S_n = 23,730,525$

▌ **Action!**

Using the Formulae

The fourth term in a geometric sequence is 24, and the ninth term is 768. What is the sum of the first 20 terms in the sequence?

find r

$$\frac{768}{24}$$

$$= 32$$

$$r = \sqrt[5]{32}$$

$$r = 2$$

find a

divide 24 by 2 three times

$$a = \frac{24}{2^3}$$

$$a = 3$$

$$S_n = \frac{3(2^{20} - 1)}{2 - 1}$$

$$S_n = 3,145,725$$

Consolidation

Unit Formulae