

What's Going On?

Checking In

Minds on

Add 'Em Up!

Action!

Arithmetic Series Formulae

Consolidation

Using The Formulae

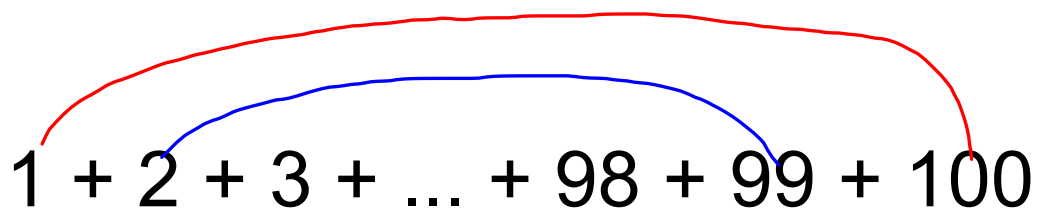
Learning Goal - I will be able to calculate the sum of an arithmetic series.

Minds on

Add 'Em Up!

Calculate the sum of the numbers from 1 to 100.

***This is not a question from last time!**

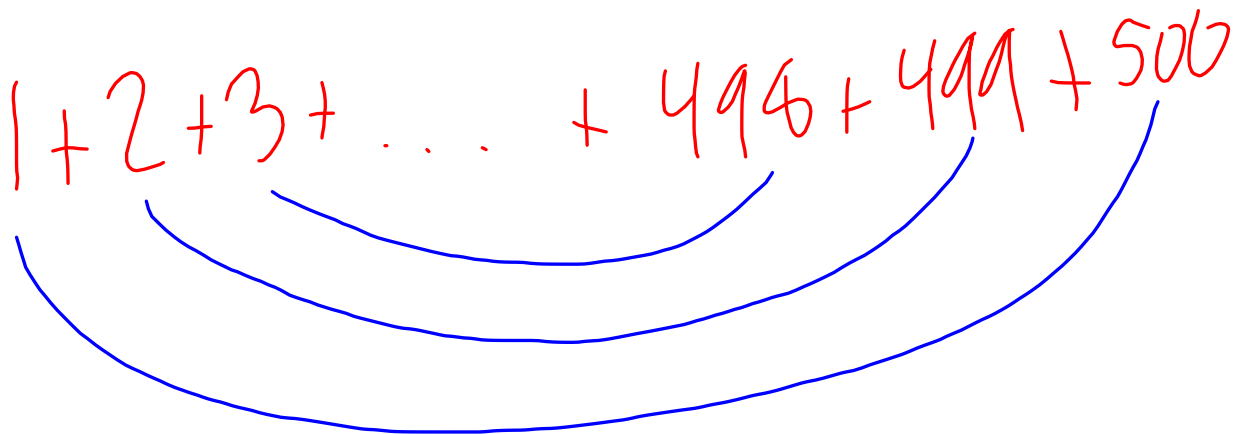

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

$$101 \times 50 = 5050$$

Minds on

Add 'Em Up!

Calculate the sum of the numbers from 1 to 500.

$$1 + 2 + 3 + \dots + 498 + 499 + 500$$
The image shows a handwritten red equation: $1 + 2 + 3 + \dots + 498 + 499 + 500$. Below the equation, there are three blue curved lines (arcs) that group the terms. The innermost arc connects 1 and 500. The middle arc connects 2 and 499. The outermost arc connects 3 and 498. This illustrates the pairing of terms in an arithmetic series to find the sum.

$$250 \times 501 \\ = 125250$$

Minds on

Add 'Em Up!

Calculate the sum of all the odd numbers from 1 to 100.

$$1 + 3 + 5 + \dots + 95 + 97 + 99$$

$$25 \times 100 = 2500$$

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

$$\underbrace{101}_{\text{purple}} \times \underbrace{50}_{\text{green}} = 5050$$

\uparrow $\frac{n}{2}$

$$1 + 2 + 3 + \dots + 498 + 499 + 500$$

$$\underbrace{250}_{\text{green}} \times \underbrace{501}_{\text{purple}} = 125250$$

\uparrow $\frac{n}{2}$

$$1 + 3 + 5 + \dots + 95 + 97 + 99$$

$$\underbrace{25}_{\text{green}} \times \underbrace{100}_{\text{purple}} = 2500$$

\uparrow $\frac{n}{2}$

Minds on

Sequences vs. Series

Series

The sum of the terms of a sequence.

Partial Sum

The sum, S_n , of the first n terms of a sequence

Action!

Sum of an Arithmetic Series

$$S_n = a + (a+d) + (a+2d) + \dots + \underline{a+(n-2)d} + a+(n-1)d$$

$$S_n = \underline{a+(n-1)d} + a+(n-2)d + \dots + (a+d) + a$$

$$2S_n = 2a + (n-1)d + \dots + 2a + (n-1)d$$

$$\frac{2S_n}{2} = \frac{n[2a + (n-1)d]}{2}$$

Action!

Sum of an Arithmetic Series

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

$a + a$ (handwritten blue)

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$t_1 + t_n$ (handwritten red)

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$a + (n-1)d$ (handwritten red)

Action!

The Formulae

Arithmetic

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

Consolidation

Using the Formulae

In an amphitheatre, seats are arranged in 50 semicircular rows. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the theatre?

$$d = 4$$

$$a = 23$$

Method 1

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

$$= \frac{50[2(23) + (50-1)(4)]}{2}$$

$$= 25(46 + 196)$$

$$= 25(242)$$

$$= 6050$$

Consolidation

Using the Formulae

In an amphitheatre, seats are arranged in 50 semicircular rows. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the theatre?

$$d=4$$

$$a=23$$

Method 2

find t_n first

$$t_n = 23 + (50-1)(4)$$

$$t_n = 23 + 196$$

$$t_n = 219$$

$$S_n = \frac{50(23 + 219)}{2}$$

$$= 25 \times 242$$

$$= 6050$$

Consolidation

Using the Formulae

Determine the sum of

- 31, - 35, - 39, ... - 403

$$a = -31 \quad t_n = -403$$

$$d = -4 \quad n = ?$$

$$t_n = a + (n-1)d$$

$$-403 = -31 + (n-1)(-4)$$

$$+31 \quad +31$$

$$\frac{-372}{-4} = \frac{(n-1)(-4)}{-4}$$

$$93 = n - 1$$

$$n = 94$$

$$S_n = \frac{94(-31 - 403)}{2}$$

$$S_n = 47 \times (-434)$$

$$S_n = -20398$$