Introduction to Sequences and Series

Arithmetic and Geometric Sequences

Definitions

Sequence: An ordered list of numbers.

Examples: 635, 630, 625, 620, 615 …

128, 64, 32, 16, 8 …

Term: A number in a sequence. Subscripts are usually used to identify the positions of the terms.

Examples: $t_1 = 635$, $t_2 = 630$, $t_3 = 625$ …

Examples: $t_1 = 128$, $t_2 = 64$, $t_3 = 32$ …

Arithmetic Sequence: A sequence that has the same difference, the **common difference**, between any pair of consecutive terms.

Examples: 6, 8, 10, 12, 14 …

150, 141, 132, 123, 114 …

Recursive Sequence: A sequence for which one term (or more) is given and each successive term is determined from the previous term(s).

Examples: 6, 6 + 2, (6 + 2) + 2, (6 + 2 + 2) +2 …

a, a + d, (a + d) + d, (a + d + d) + d …

General Term: A formula, labelled t_n, that expresses each term of a sequence as a function of its position. For example, if the general term is $t_n = 2n$, then to calculate the 12th term (t_{12}), substitute $n = 12$.

Examples: $t_n = 2n + 4$ OR $t_n = 6 + (n – 1)2$

$\mathbf{t_n = a + (n – 1)d}$, a represents the first term, d represents the difference between successive terms

Recursive Formula: A formula relating the general term of a sequence to the previous term(s).

Examples: $t_1 = 6$, $t_n = t_{n-1} + 2$, where $n > 1$

$\mathbf{t_1 = a, t_n = t_{n-1} + d}$, where $n > 1$

Examples

1. Decide whether the given sequence is algebraic. Then, determine the general term and the recursive formula for the sequence. Use your formulae to determine the 10^{th} term of the sequence.

a. 6, 15, 24, 33, 42 …

The terms in the sequence increase by 9, therefore this is an **arithmetic sequence**.

The formula for the general term of an arithmetic sequence is $\mathbf{t_n = a + (n - 1)d}$.
For this example, a = 6 and d = 9.
Therefore, the general term for this example is $\mathbf{t_n = 6 + (n - 1)9}$ OR $\mathbf{t_n = 9n - 3}$

The generalized recursive formula for an arithmetic sequence is $t_1 = a$, $t_n = t_{n-1} + d$, where $n > 1$.
Therefore, the recursive formula for this example is $\mathbf{t_1 = 6, t_n = t_{n-1} + 9}$**, where** $\mathbf{n > 1}$.

The 10^{th} term in this sequence is $\mathbf{t_n = 6 + ((10) - 1)9 = 6 + (9)9 = 6 + 81 = \underline{87}}$.

Sequence	General Term	Expanded General Term	Recursive Formula	10^{th} Term
1, 5, 9, 13, 17 …				
8, 11, 14, 17, 20 …				
28, 19, 10, 1, -8 …				
784, 588, 392 …				