What's Going On?

Checking In

Minds on Identities you know

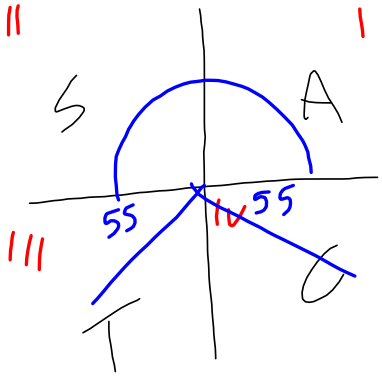
Action! Identities you don't

Consolidation Simplifying and Proving by Factoring

Learning Goal - I will be able to prove trigonometric identities

L.G.L.

If sin θ = -0.8190 and θ is between 0° and 360°, determine all possible values of θ to the nearest degree.



$\sin\beta = 0,8190$

$\beta \doteq 55°$

$\theta = 235°$ and $305°$

Minds on

Identities

An identity is a mathematical statement that is true for all values of the given variables.

If the identity involves fractions, the denominators cannot be zero. Any restrictions on a variable must be stated.

Minds on

Identities You Know

Reciprocal Identities

$$\csc\theta = \frac{1}{\cos\theta} \qquad \sec\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

sin cos

We'll accept these as definitions.

Minds on

Identities You Know

The Basic "Identities"

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

We'll accept these as definitions.

Action!

Identities You Don't

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Everything in mathematics is built upon a relatively small set of definitions.

Anything that is then introduced must be proven to be accepted.

Prove it!

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

L.S.

$\tan\theta$

$= \frac{y}{x}$ by basic identity

R.S.

$\frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{r} \cdot \frac{r}{x}$

$= \frac{y}{x}$

L.S. = R.S. $\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$

Action!

Identities You Don't

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Your Turn!

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

L.S.

$\cot\theta$

$= \frac{1}{\tan\theta}$ by reciprocal

$= \frac{1}{\frac{y}{x}}$

$= \frac{x}{y}$

R.S.

$\frac{\cos\theta}{\sin\theta}$

$= \frac{\frac{x}{r}}{\frac{y}{r}}$

$= \frac{x}{r} \times \frac{r}{y}$

$= \frac{x}{y}$

L.S. = R.S. $\therefore \cot\theta = \frac{\cos\theta}{\sin\theta}$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

Prove it!

*sin²θ is just (sin θ)²

$$\sin^2\theta + \cos^2\theta = 1$$

L.S.

$\sin^2\theta + \cos^2\theta$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2 + y^2}{r^2}$

$= \frac{r^2}{r^2}$ by Pythagorean Theorem

$= 1$

R.S.

1

L.S. = R.S.

$\therefore \sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

Prove it!

$$1+\tan^2\theta=\sec^2\theta$$

L.S.

$1+\tan^2\theta$

$=1+\frac{\sin^2\theta}{\cos^2\theta}$

$=\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$

$=\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}$

$=\frac{1}{\cos^2\theta}$ by Pythagorean Identity

R.S.

$\sec^2\theta$

$=\frac{1}{\cos^2\theta}$

Action!

Identities You Don't

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Your Turn!

$$1+\cot^2\theta=\csc^2\theta$$

L.S.

$1+\cot^2\theta$

$=1+\frac{1}{\tan^2\theta}$

$=1+\frac{1}{\frac{\sin^2\theta}{\cos^2\theta}}$

$=1+\frac{\cos^2\theta}{\sin^2\theta}$

$=\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$

$=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$

$=\frac{1}{\sin^2\theta}$ by Pythagorean Identity

R.S.

$\csc^2\theta$

$=\frac{1}{\sin^2\theta}$

Consolidation

Simplifying Trigonometric Expressions

Often, when proving an identity, our first step is to simplify a complicated expression into something less complex.

Typically, we rewrite any reciprocal ratios and ratios involving tangent, as sine and cosine.

Simplify

a) $1+\cot^2\theta$

$= 1+\frac{1}{\tan^2\theta}$

$= 1+\frac{1}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= 1+\frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$ by Pythagorean Identity

b) $\frac{\cot\theta}{\cos\theta}$

$= \frac{\frac{1}{\tan\theta}}{\cos\theta}$

$= \frac{1}{\tan\theta}\times\frac{1}{\cos\theta}$

$= \frac{1}{\frac{\sin\theta}{\cos\theta}}\times\frac{1}{\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}\times\frac{1}{\cos\theta}$

$= \frac{1}{\sin\theta}$

c) $(1-\cos\theta)(1+\cos\theta)$

$= 1 + \cos\theta - \cos\theta - \cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$ by Pythagorean Identity

d) $\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$

$= \dfrac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)}$

$= \sin\theta + \cos\theta$

Consolidation

Proving by Factoring

Sometimes when proving an identity, it is necessary to factor a given expression.

Common Factor

$\sin\theta\cos\theta + \sin\theta$

$= \sin\theta(\cos\theta + 1)$

$\tan^2\theta - \sin\theta\tan\theta$

$= \tan\theta(\tan\theta - \sin\theta)$

Difference of Squares

$\sin^2\theta - \tan^2\theta$

$= (\sin\theta + \tan\theta)(\sin\theta - \tan\theta)$

$\sqrt{1} - \sqrt{\sin^2\theta}$

$= (1 + \sin\theta)(1 - \sin\theta)$

$$\tan\theta = \frac{\sin\theta + \sin^2\theta}{(\cos\theta)(1+\sin\theta)}$$

L.S.

$\tan\theta$

$= \frac{\sin\theta}{\cos\theta}$

R.S.

$= \frac{\sin\theta + \sin^2\theta}{(\cos\theta)(1+\sin\theta)}$

$= \frac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

Homework Change

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