What's Going On?

Checking In

Minds on CAST

Action! Angle of Mine

Consolidation Working Backwards

Learning Goal - I will be able to evaluate trigonometric ratios for angles between 0° and 360°.

LGL

a. Sketch the given angle in standard position.

b. Use the sketch to determine the **exact** value of the given trigonometric ratio.

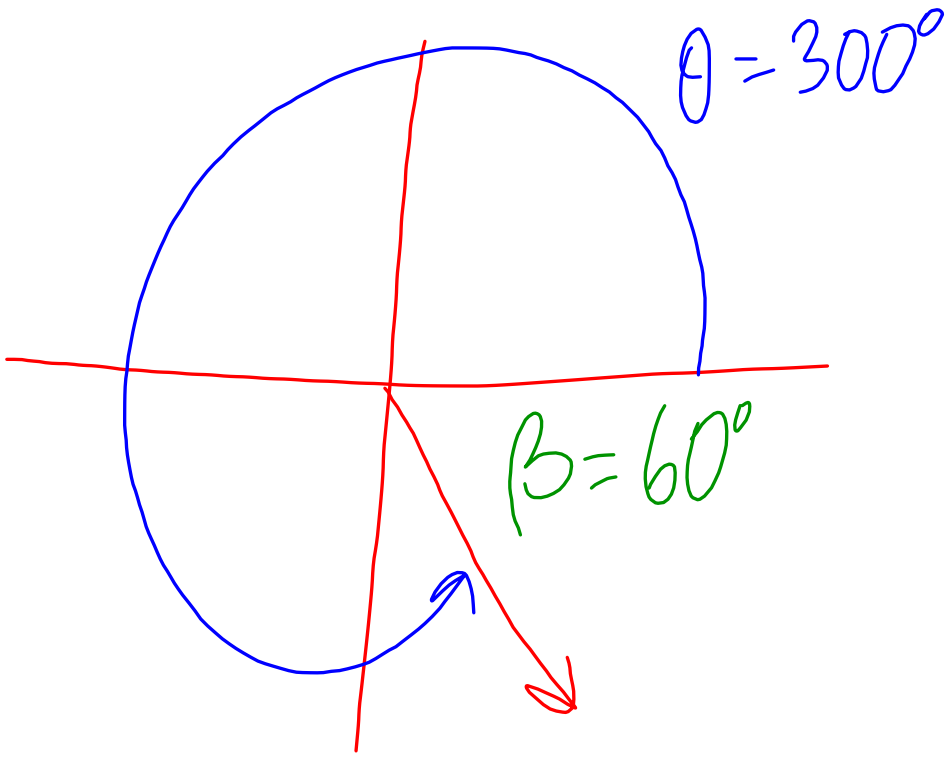
c. State all values of θ that have the same given trigonometric ratio.

$$\tan 300^{o}$$

LGL

a. Sketch the given angle in standard position.

tan 300°

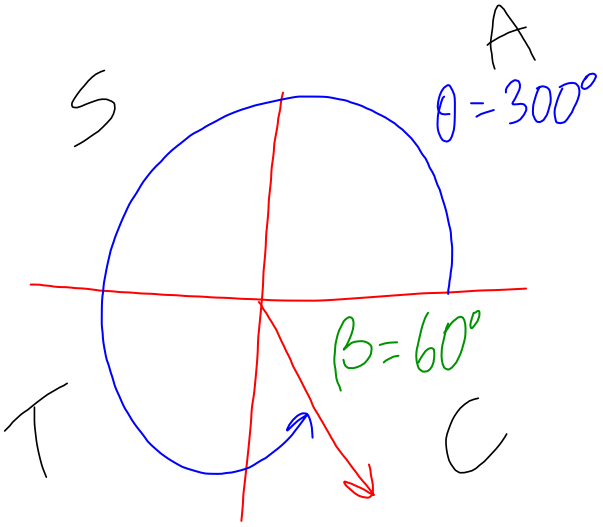


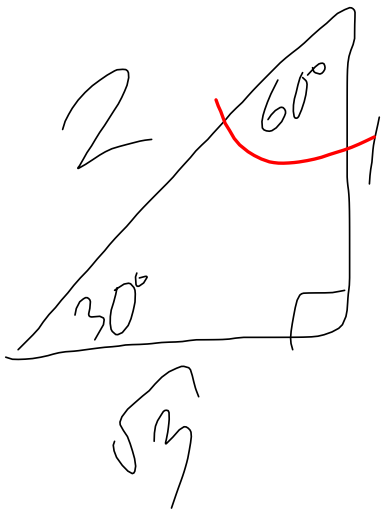
LGL

b. Use the sketch to determine the **exact** value of the given trigonometric ratio.

tan 300°

soh cah toa



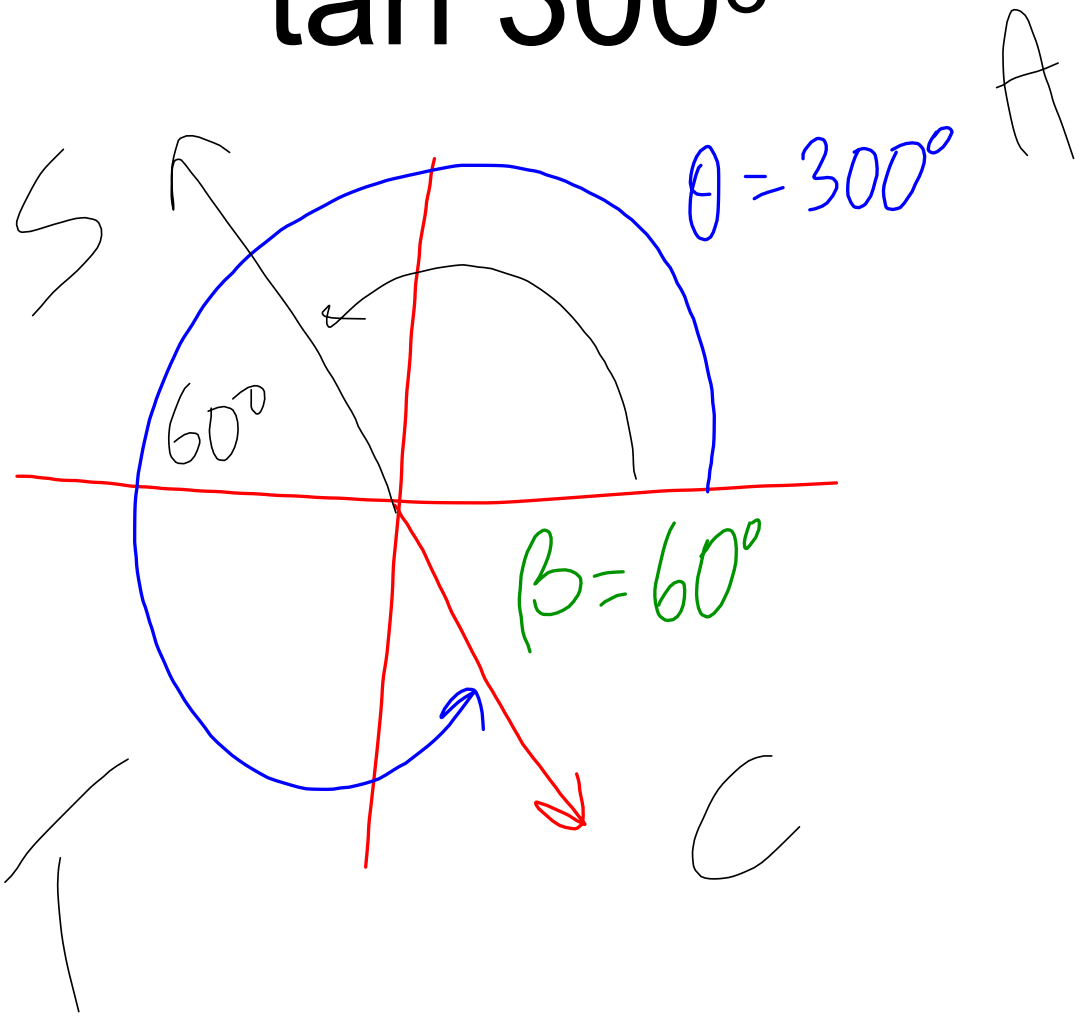


$\tan 300° = -\sqrt{3}$

LGL

c. State all values of θ that have the same given trigonometric ratio.

tan 300°



$\tan 300 = \tan 120$

Minds on

The CAST Rule

Use the CAST Rule to determine the sign of each ratio.

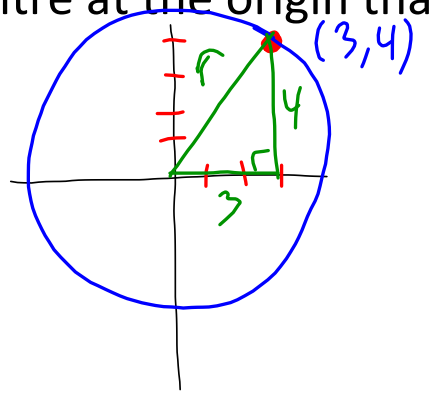
sin (150°) + cos (222°) − tan (75°) +

sec (315°) + cot (95°) − csc (200°) −

Action!

Angle of Mine

1) Sketch a circle with its centre at the origin that goes through the point P(3, 4).



a) Determine the radius of the circle

$r^2 = 3^2 + 4^2$

$r^2 = 9 + 16$

$r^2 = 25$

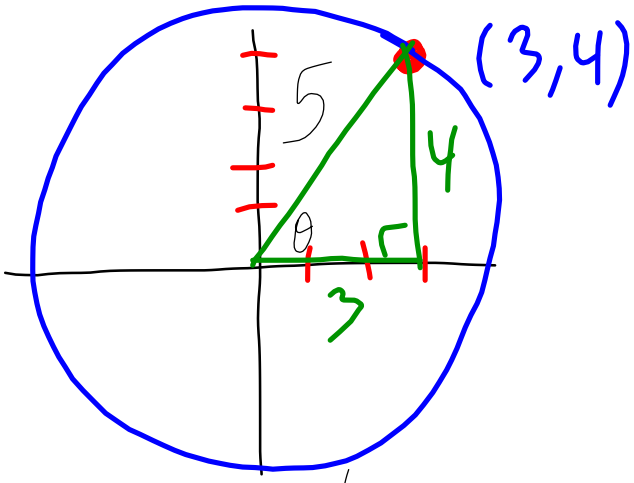
$r = 5$

b) Determine the primary trig ratios for the principal angle.
c) Determine the principal angle to the nearest degree.

Action!

Angle of Mine

b) Determine the primary trig ratios for the principal angle.



$\sin\theta = \frac{4}{5}$ | $\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

c) Determine the principal angle to the nearest degree.

$\sin\theta = \frac{4}{5}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\cos\theta = \frac{3}{5}$

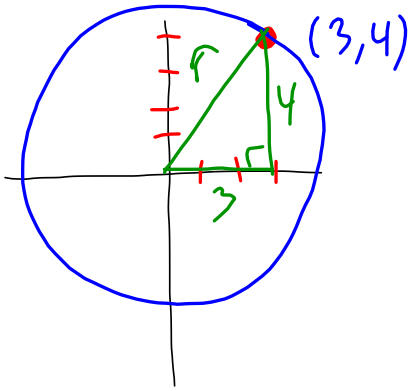
$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

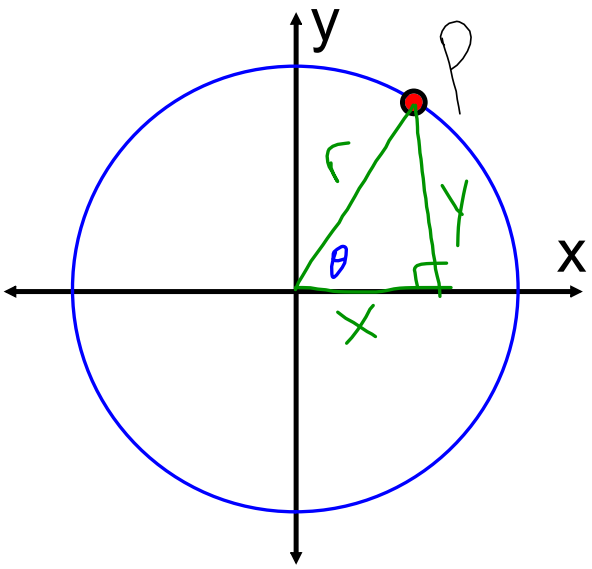
Action!

Angle of Mine



c) Determine the principal angle to the nearest degree.

For any point P(x, y) in the Cartesian plane, the trigonometric ratios for angles in standard position can be expressed in terms of x, y and r.



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

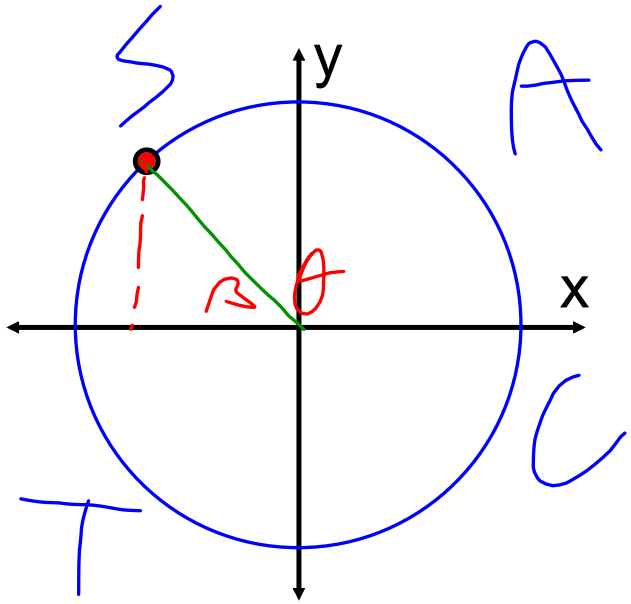
$$\tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y}$$

$$\sec\theta = \frac{r}{x}$$

$$\cot\theta = \frac{x}{y}$$

Quadrant II



$$\sin\theta = \frac{y}{r}$$

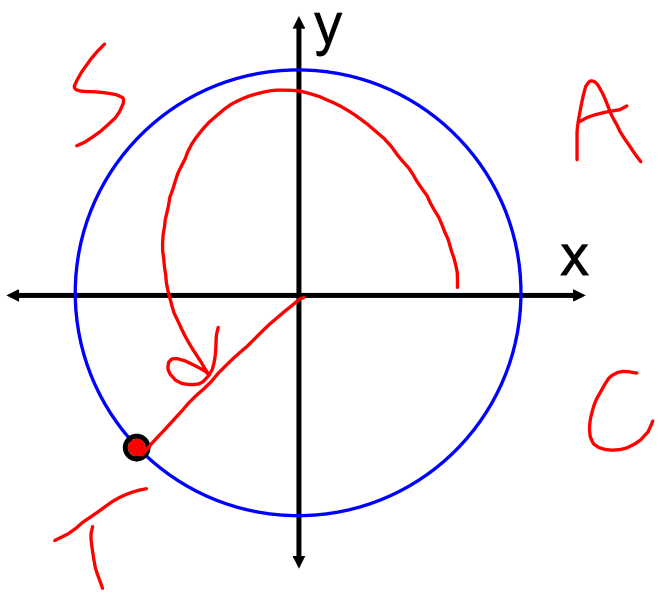
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

x is negative

y is positive

Quadrant III



$$\sin\theta = \frac{y}{r}$$

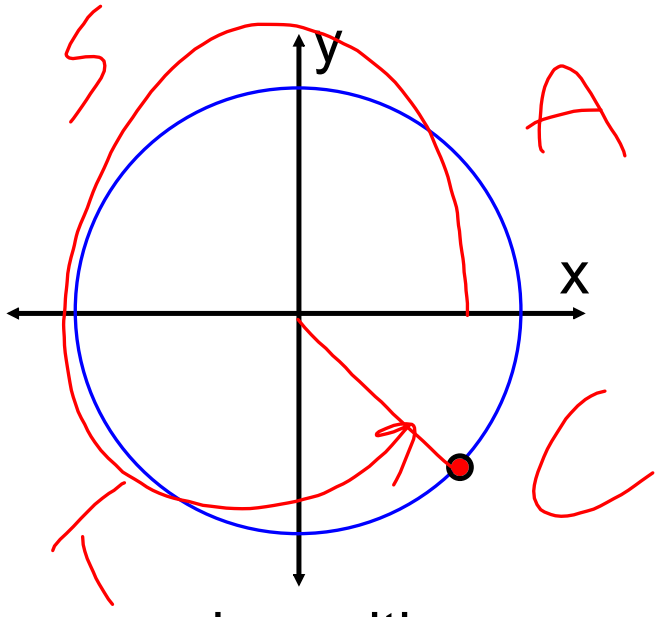
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

x is negative

y is negative

Quadrant IV



x is positive

y is negative

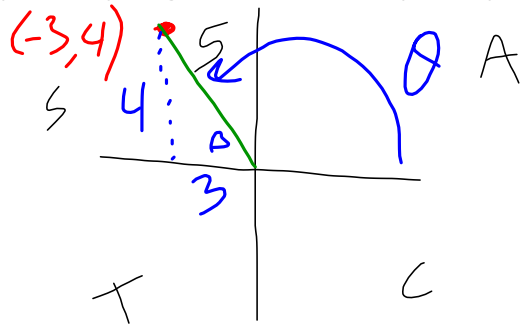
$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

2) Now, choose the point P(-3, 4) on the circumference of the circle.

a) Determine the primary trig ratios for the principal angle.



$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{-3}{5}$ $\tan\theta = \frac{4}{-3}$

b) Determine the principal angle to the nearest degree.

$\sin\beta = \frac{4}{5}$

$\beta = 53°$

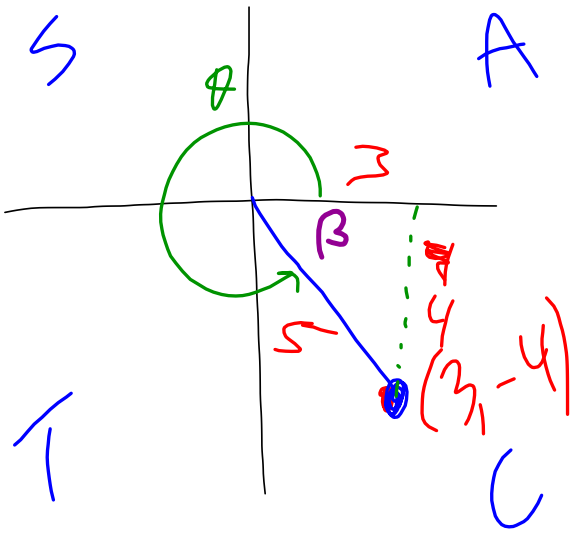
$\theta = 180 - 53$

$\theta = 127°$

2) Now, choose the point P(3, -4) on the circumference of the circle.

a) Determine the primary trig ratios for the principal angle.

b) Determine the principal angle to the nearest degree.



$\sin\theta = \frac{-4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{-4}{3}$

$\sin\beta = \frac{4}{5}$ $\cos\beta = \frac{3}{5}$ $\tan\beta = \frac{4}{3}$

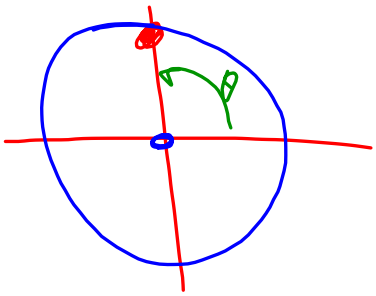
$\beta \doteq 53°$

$\theta = 360° - \beta$

$\theta \doteq 307°$

Example 1: Sketch a circle with its centre at the origin that goes through the point P(0, 1).

Determine the radius of the circle.



$r = 1$

Determine the primary trig ratios for the principal angle.

$\sin\theta = 1$ $\cos\theta = 0$ $\tan\theta =$ undefined

Determine the principal angle to the nearest degree.

$\theta = 90°$

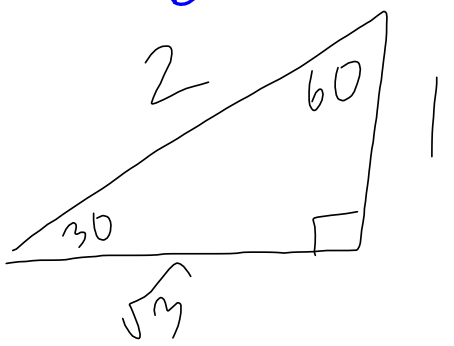
Consolidation

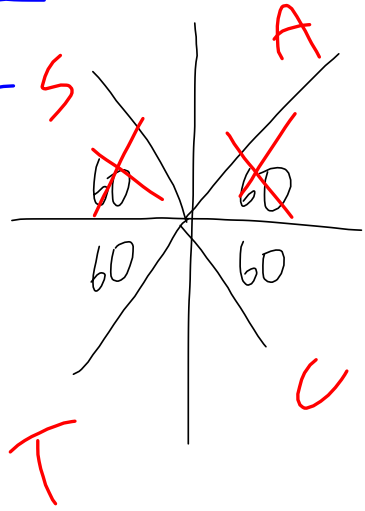
Working Backwards

Example 2: Determine the values of θ if $csc\theta = -\frac{2\sqrt{3}}{3}$ and $0^o \leq \theta \leq 360^o$.

If $csc\theta = -\frac{2\sqrt{3}}{3}$ then $\sin\theta = -\frac{3}{2\sqrt{3}}$

$\frac{-3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$





Consolidation

Working Backwards

Example 2: Determine the values of θ if $\csc\theta = -\frac{2\sqrt{3}}{3}$ and $0^\circ \le \theta \le 360^\circ$.

if $\csc\theta = \frac{-2\sqrt{3}}{3}$

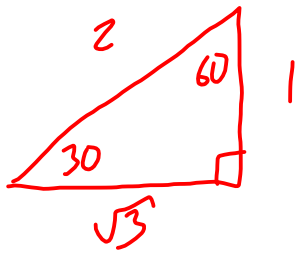
$\sin\theta = \frac{-3}{2\sqrt{3}}$ rationalize the denominator!

$\frac{-3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}}{2(3)}$

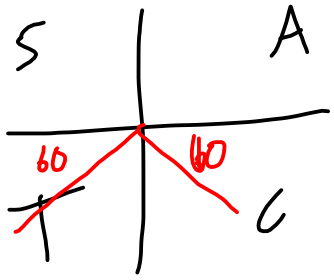
$= \frac{-3\sqrt{3}}{6}$

$= -\frac{\sqrt{3}}{2}$

if $\sin\theta = -\frac{\sqrt{3}}{2}$ ...



$\theta = 60^\circ$ where sine is negative



$\theta = 240^\circ$ and 300°