What's Going On?

Checking In

Minds on Angles in Standard Position

Action! Exploring Angles Greater than 90°

Consolidation Finding Equivalent Ratios

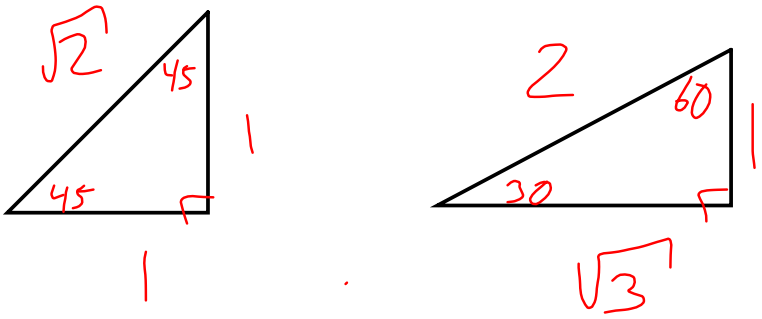
Learning Goal - I will be able to identify angles that share related trigonometric ratios.

L.G.L.

Determine the exact value of:

$$\sin(30) + \sec(45) - \tan(60)$$

using special triangles.



$$= \frac{1}{2} + \sqrt{2} - \sqrt{3}$$

$$= \frac{1}{2} + \frac{2\sqrt{2}}{2} - \frac{2\sqrt{3}}{2}$$

$$= \frac{1 + 2\sqrt{2} - 2\sqrt{3}}{2}$$

Minds on

A Few Terms

standard position

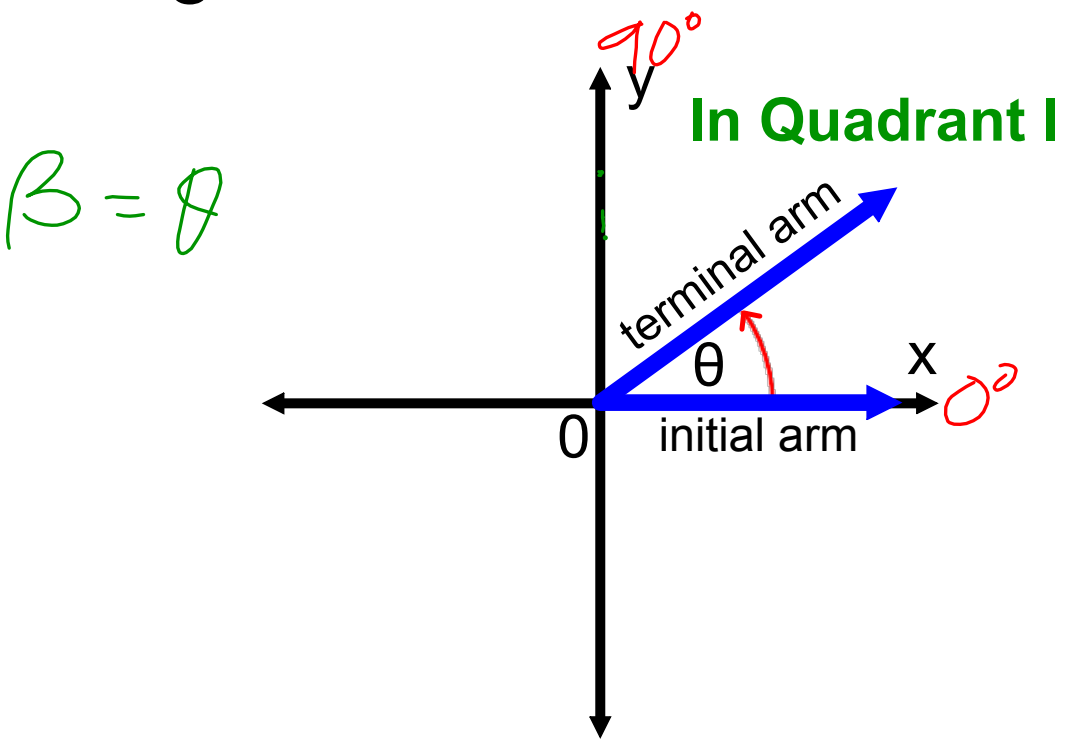
an angle in the Cartesian plane whose vertex lies at the origin and whose initial arm lies on the positive x-axis. Angle θ is measured from the initial arm to the terminal arm.

principal angle (θ)

the counter clockwise angle between the initial arm and the terminal arm of an angle in standard position. Its value is between 0° and 360°.

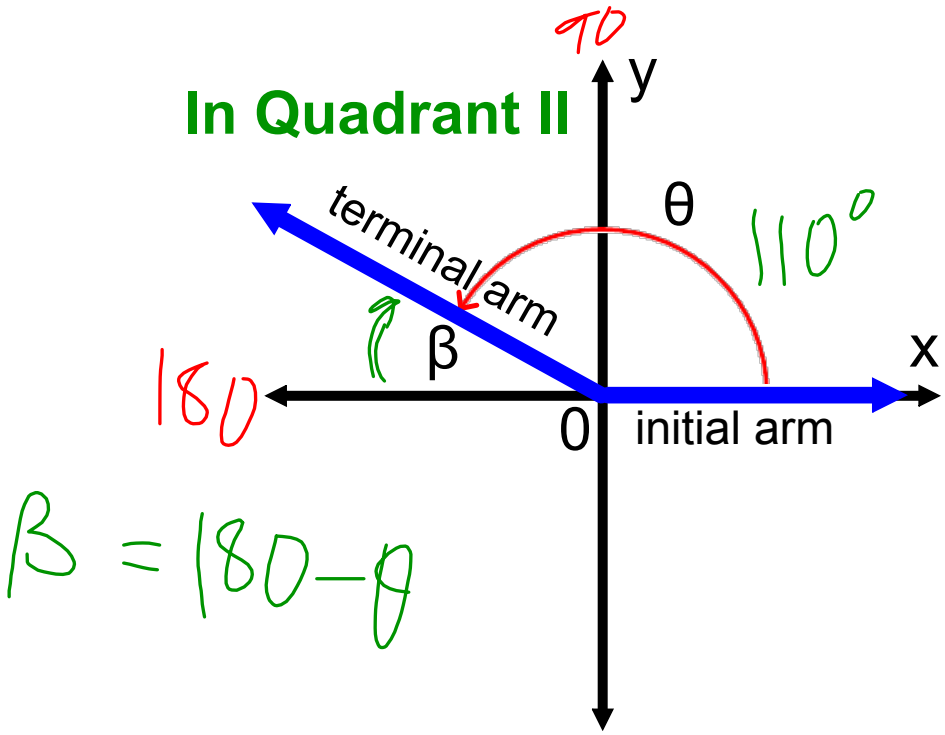
Minds on

Angles in Standard Position



Minds on

Angles in Standard Position



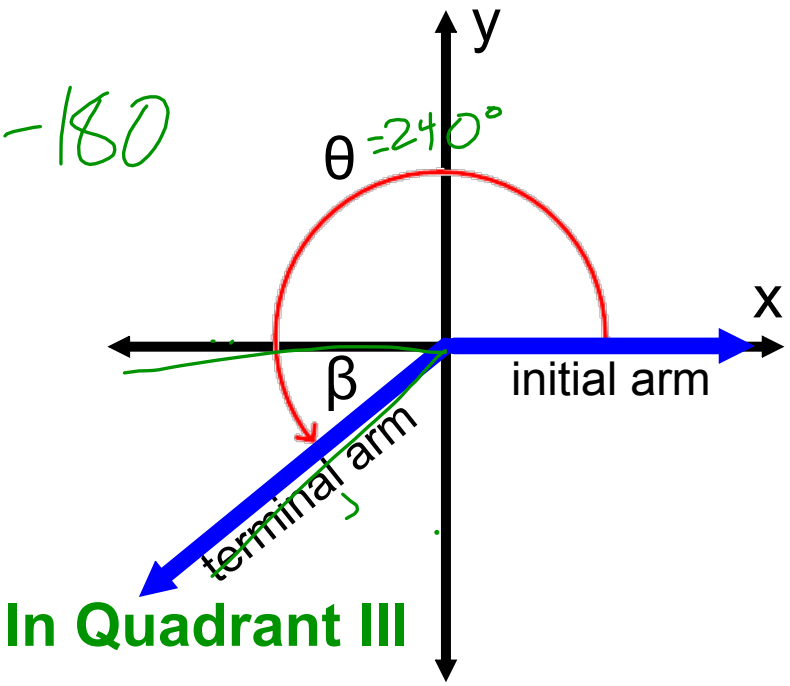
related acute angle (β)

the acute angle between the terminal arm of an angle in standard position and the x-axis when the terminal arm lies in quadrants 2, 3, or 4. Its value is between 0° and 90°.

Minds on

Angles in Standard Position

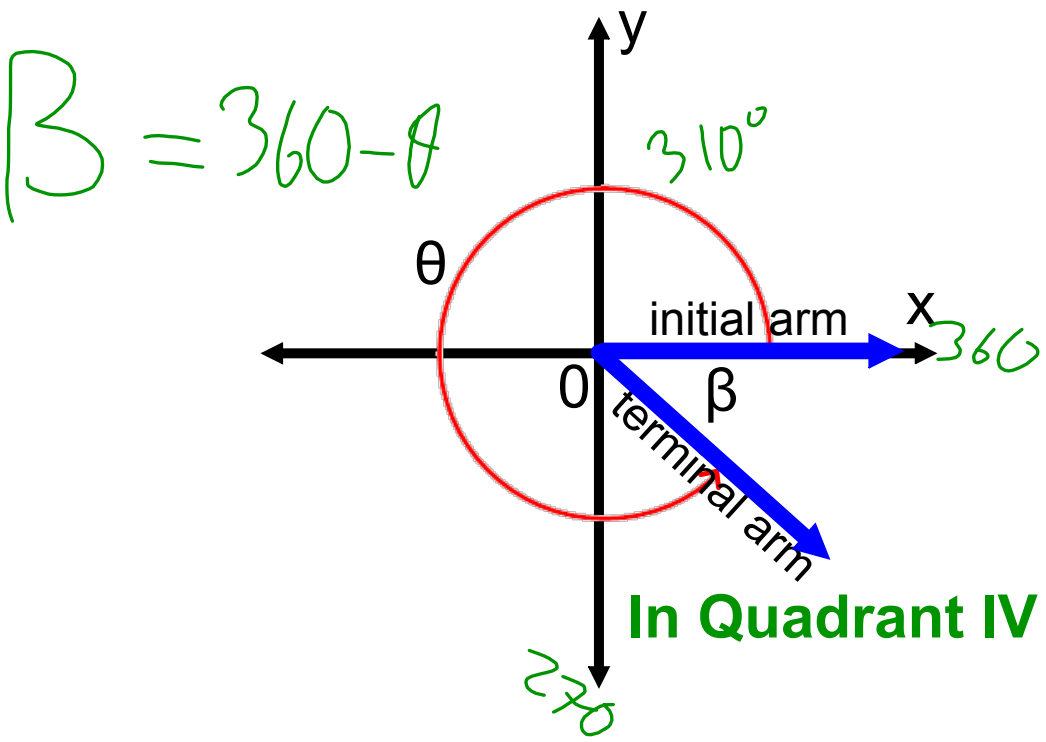
$\beta = \theta - 180$



In Quadrant III

Minds on

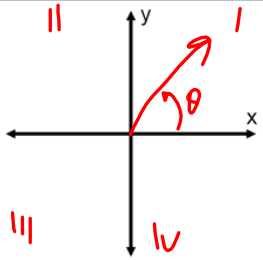
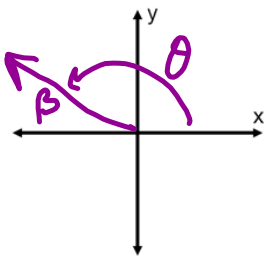
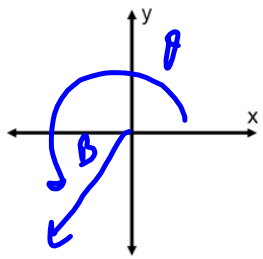
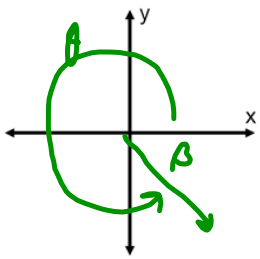
Angles in Standard Position



Action!

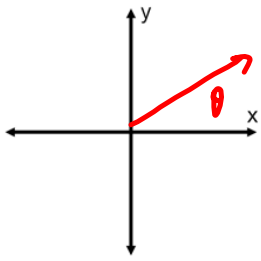
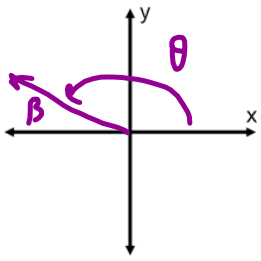
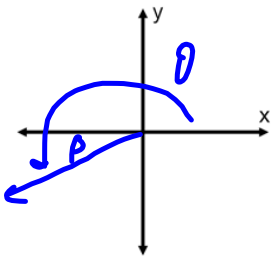
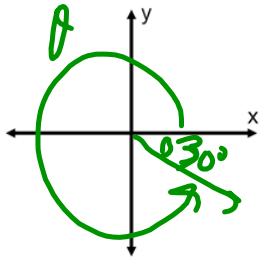
Exploring Angles Greater than 90°

1. Sketch the angles given in the space provided in standard form. Be sure to include the principal angle and the related acute angle for each. Then complete the table that follows.

Angle	45°	135°	225°	315°
Sketch				

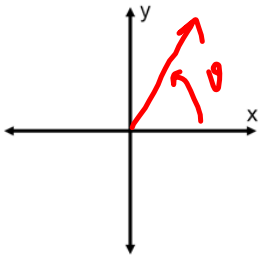
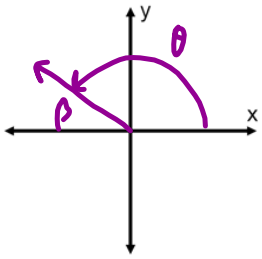
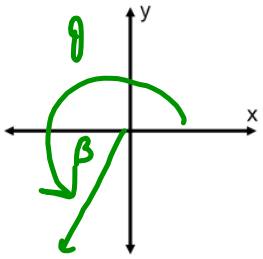
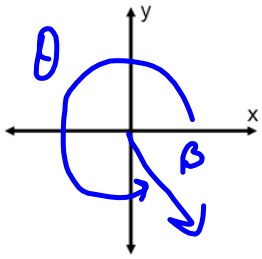
Principal Angle θ	Quadrant #	Related Acute Angle β	$\sin\theta$	$\cos\theta$	$\tan\theta$
45°	1	45°	0.707	0.707	1
135°	2	45°	0.707	−0.707	−1
225°	3	45°	−0.707	−0.707	1
315°	4	45°	−0.707	0.707	−1

2. Sketch the angles given in the space provided in standard form. Be sure to include the principal angle and the related acute angle for each. Then complete the table that follows.

Angle	30°	150°	210°	330°
Sketch				

Principal Angle θ	Quadrant #	Related Acute Angle β	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	1	30°	0.5	0.866	0.577
150°	2	30°	0.5	-0.866	-0.577
210°	3	30°	-0.5	-0.866	0.577
330°	4	30°	-0.5	0.866	-0.577

3. Sketch the angles given in the space provided in standard form. Be sure to include the principal angle and the related acute angle for each. Then complete the table that follows.

Angle	60°	120°	240°	300°
Sketch				

Principal Angle θ	Quadrant #	Related Acute Angle β	$\sin\theta$	$\cos\theta$	$\tan\theta$
60°	1	60°	0.866	0.5	1.732
120°	2	60°	0.866	-0.5	-1.732
240°	3	60°	-0.866	-0.5	1.732
300°	4	60°	-0.866	0.5	-1.732

Trigonometric Ratio	Quadrant			
	I	II	III	IV
sine	+	+	−	−
cosine	+	−	−	+
tangent	+	−	+	−

CAST tells us what trig ratios are positive in what quadrants.

All ratios are positive in quadrant I

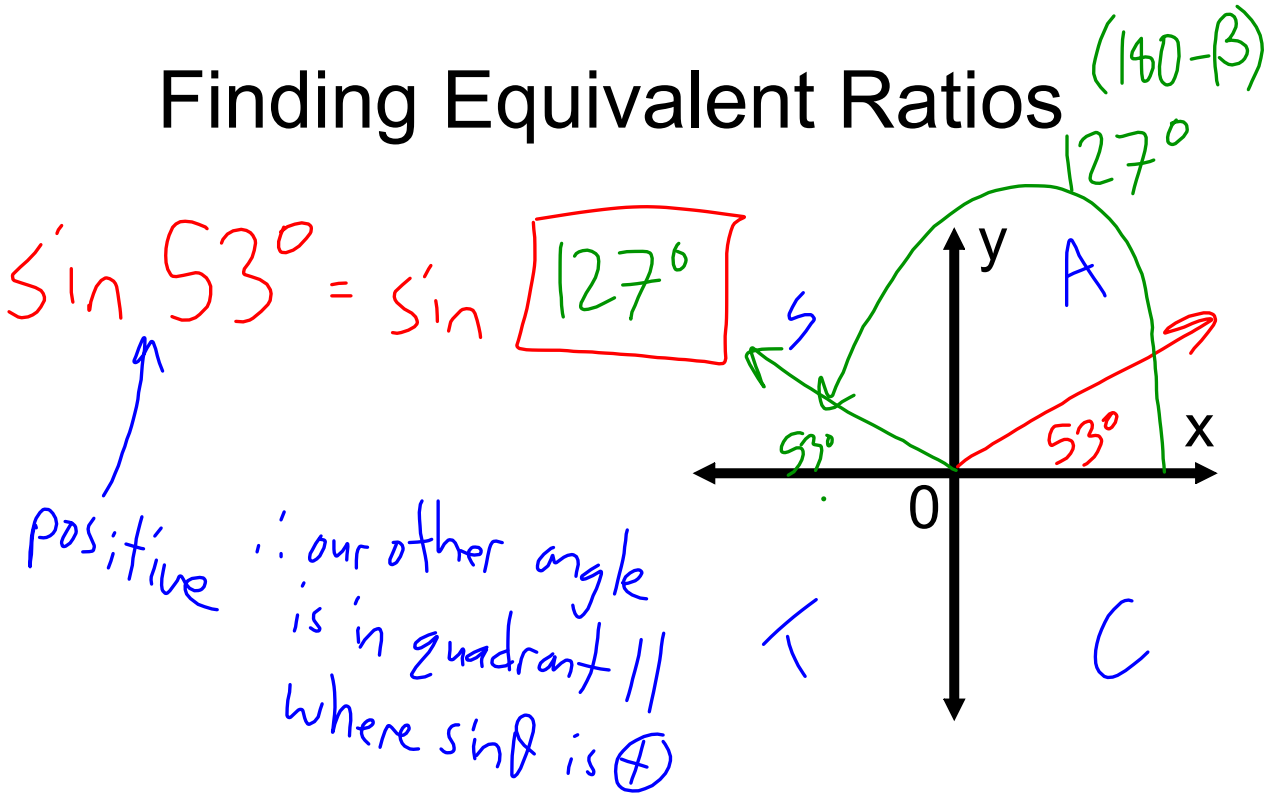
Sin is positive in quadrant II

Tan is positive in quadrant III

Cos is positive in quadrant IV

Consolidation

Finding Equivalent Ratios



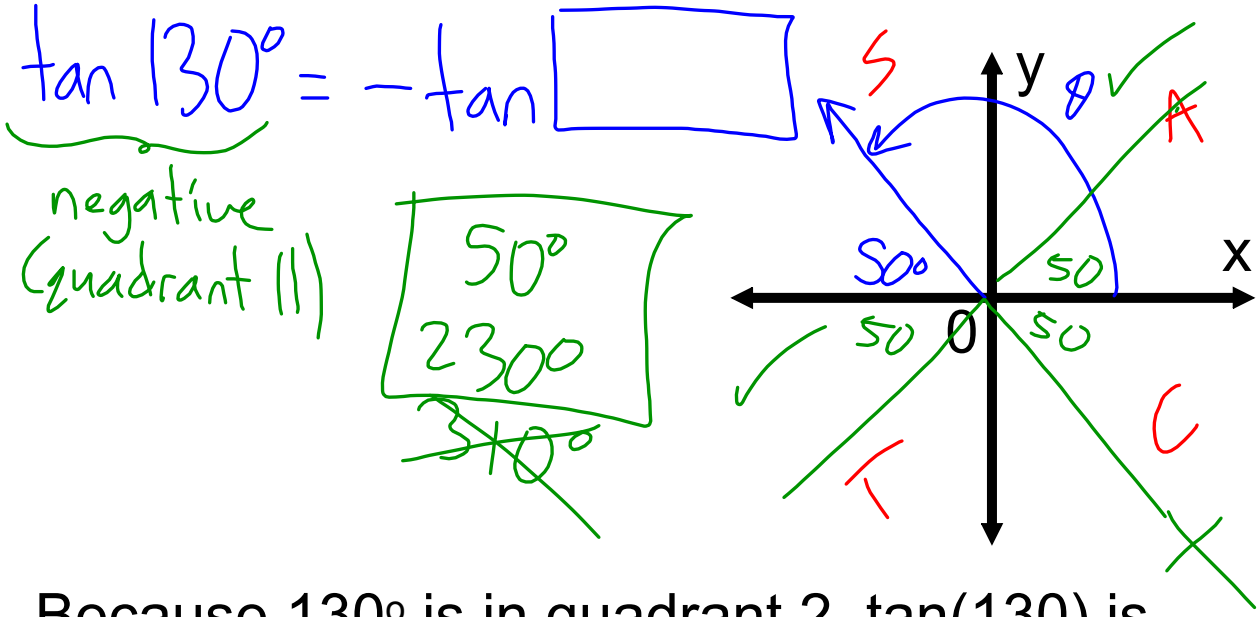
sin(53) is positive (in quadrant 1)

We need to find the other quadrant in which sine is positive (quadrant 2)

So, our answer is the angle in quadrant 2 with a related acute angle of 53.

Consolidation

Finding Equivalent Ratios



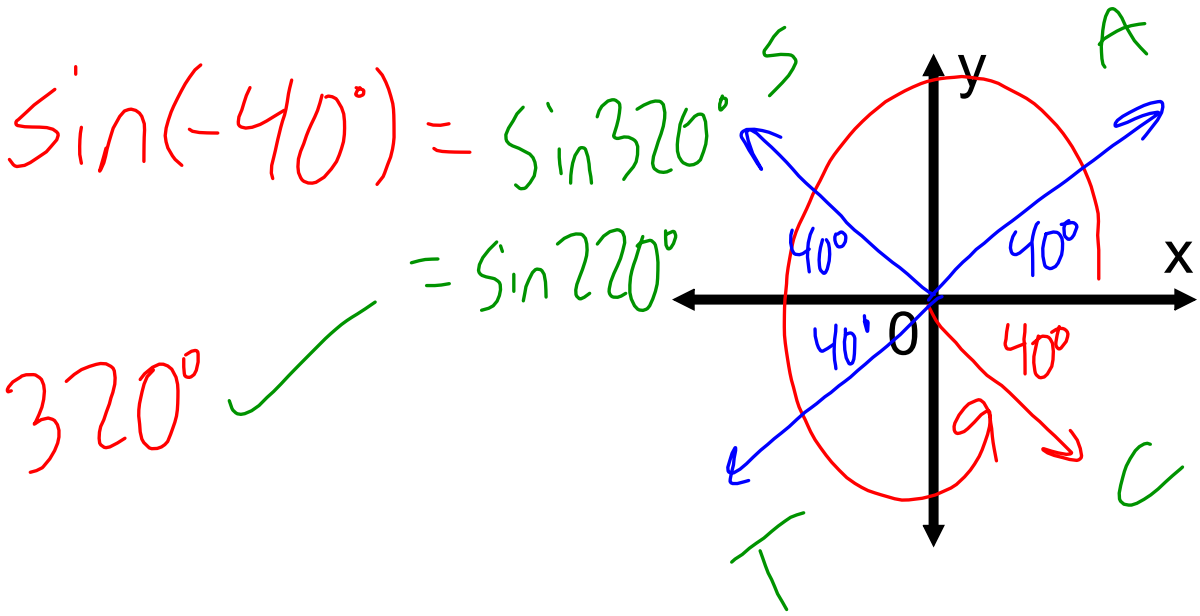
Because 130° is in quadrant 2, tan(130) is negative.

Now, because we are looking to find values of **-tan( )**, the values of tan we look at must originally be positive. This happens in quadrants 1 and 3.

The angles we are interested in will have the same related acute angle (50°).

Consolidation

Finding Equivalent Ratios



*Negative angles are measured from the positive x-axis in the clockwise direction (opposite direction of positive angles)

So -40° is the same as 320°. So our original angle is in quadrant IV

In quadrant 4, sine is negative, so our other answer is in quadrant III.

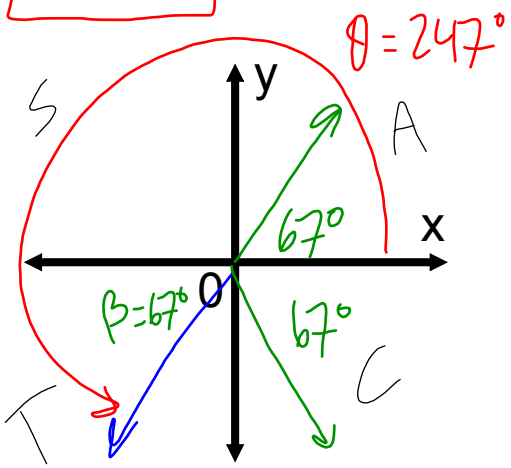
We use the same related acute angle (40°) to find our answer.

Consolidation

Finding Equivalent Ratios

$-\cos(-113°) = \cos\square$

$-113°$ is the same as $(360-113) = 247°$



$-\cos(-113°) = \cos 67°$

$-\cos(-113°) = \cos 293°$

To tackle this question, first we determine the value of -113° as a positive angle.

To do this we can do 360 - 113 = 247.

Now, we are talking about the negative of cos 247. Because 247 is in quadrant 3 (with a related acute angle of 67), we know that cos(247) is a negative value. However, we are talking about -cos(247) which will be the negative of a negative (positive).

So we need to find where cos is positive (quadrants 1 and IV).

Our answers, then, are the angles in quadrants 1 and 4 with related acute angles of 67.