What's Going On?

Checking In	Your Thoughts...
Minds on	3 Forms
Action!	Profit Functions and Systems
Consolidation	Questions from the homework?

Learning Goal - I will be ready for tomorrow's test!

Determine the number of points of intersection of the parabola **$f(x) = -2x^2 - 5x + 7$** and the line **$g(x) = -7x + 1$**.

$$-2x^2 - 5x + 7 = -\cancel{7x} + \cancel{1} \quad 0$$
$$+7x - 1 \qquad +7x - 1$$

$$-2x^2 + 2x + 6 = 0$$

$$b^2 - 4ac = (2)^2 - 4(-2)(6)$$
$$= 4 + 46$$
$$= 52$$

Minds on

	Vertex Form $y = a(x-h)^2 + k$
Vertex	How to: **It's right in there!**
	Example: $y = \frac{1}{4}(x+6)^2 - 4$ **(-6, -4)**

Minds on

<table>
<tr><td></td><td>Vertex Form
$y = a(x-h)^2 + k$</td></tr>
<tr><td rowspan="2">Zeros</td><td>How to:
Expand it into standard form.
Then either factor or use the quadratic formula.
ORRRR "Solve for x!"</td></tr>
<tr><td>Example: $y = \frac{1}{4}(x+6)^2 - 4$
$0 = \frac{1}{4}(x+6)^2 - 4$
$4 = \frac{1}{4}(x+6)^2$
$\sqrt{16} = \sqrt{(x+6)^2}$
$\pm 4 = x+6$
$x = 4-6$ or
$x = -4-6$</td></tr>
</table>

Minds on

y-Intercept	**Vertex Form** $y = a(x-h)^2 + k$
	How to: Expand it into standard form, it's right there! Or, sub in 0 for x and solve for y.
	Example: $y = \frac{1}{4}(x+6)^2 - 4$ $y = 5$

Minds on

Standard Form

$$y = ax^2 + bx + c$$

Vertex

How to: 1. Complete the square.

2. Find the zeros by factoring or quadratic formula, then find the axis of symmetry (x-value of vertex), then plug in to find the y-value of vertex.

3. Use two symmetrical points to find axis of symmetry.

Example: $y = 2x^2 - 4x - 16$

$$y = 2(x^2 - 2x + 1 - 1) - 16$$

$$y = 2(x-1)^2 - 2 - 16$$

$$y = 2(x-1)^2 - 18$$

Minds on

Standard Form

$$y = ax^2 + bx + c$$

Zeros

How to:

Factor or use quadratic formula.

Example: $y = 2x^2 - 4x - 16$

$y = 2(x^2 - 2x - 8)$

$y = 2(x-4)(x+2)$

$\therefore x = 4$

$x = -2$

are the zeros

Minds on

y-Intercept	**Standard Form** $y = ax^2 + bx + c$
	How to: It's the c-value!
	Example: $y = 2x^2 - 4x - 16$

Minds on

	Factored Form $y = a(x - r)(x - s)$
Vertex	How to: Use zeros to find axis of symmetry (x-value of vertex), then plug in solve for y.
	Example: $y = -2(x - 2)(x + 6)$ $x=2$ $x=-6$ $\frac{2-6}{2} = -2$ $y = -2(-2-2)(-2+6)$ $y = 32$ vertex $(-2, 32)$

Minds on

<table>
<tr><td></td><td>Factored Form
$y = a(x - r)(x - s)$</td></tr>
<tr><td rowspan="2">Zeros</td><td>How to:
They're right there! Careful with the signs.</td></tr>
<tr><td>Example: $y = -2(x - 2)(x + 6)$
$x = 2$ $x = -6$</td></tr>
</table>

Minds on

y-Intercept	**Factored Form** $y = a(x-r)(x-s)$
	How to: 1. Expand it into standard form, it will be the constant. 2. Multiply a, r and s! ARS!
	Example: $y = -2(x-2)(x+6)$ $y = 24$

Action!

Profit Functions

$\cdot x$

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

a. The revenue function, $R(x)$.

$R(x) = x(-2x+20)$

$= -2x^2 + 20x$

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

b. The maximum revenue, **by completing the square**.

$R(x) = -2x^2 + 20x$

$= -2(x^2 - 10x)$

$= -2(x^2 - 10x + 25 - 25)$

$= -2(x - 5)^2 + 50$

$\therefore$ the max revenue is \$50,000!

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

c. The number of units that need to be sold to achieve the maximum revenue.

5,000

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

d. The profit function, $P(x)$.

$$\text{Profit} = R(x) - C(x)$$
$$= -2x^2 + 20x - (2x+16)$$
$$= -2x^2 + 20x - 2x - 16$$
$$= -2x^2 + 18x - 16$$

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = \cancel{2x} + 16$, in thousands of dollars, determine:

e. The start-up cost.

$P(x) = -\cancel{2x^2} + \cancel{18x} - 16$

$16,000

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

f. The break-even points, **by factoring**.

$$P(x) = -2x^2 + 18x - 16$$
$$= -2(x^2 - 9x + 8)$$
$$= -2(x-1)(x-8)$$

$\therefore$ we break even after selling 1,000 units and again at 8,000.

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

g. The maximum profit, **using the break-even points**.

$P(x) = -2(x-1)(x-8)$

$\frac{1+8}{2}$

axis of symmetry $= \boxed{4.5}$

So max profit $= -2(x-1)(x-8)$

$= -2(3.5)(-3.5)$

$= -2(-12.25)$

$= 24.5$

$\therefore$ max profit is \$24,500.

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

h. The number of units that need to be sold to achieve the maximum profit.

The zeros are 1 and 8

so we need to sell

$\frac{1+8}{2} = 4.5$

4,500 units

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

i. The number of units that need to be sold to reach a profit of $\boxed{\$12,000}$.

$$P(x) = -2x^2 + 18x - 16$$

$$\cancel{12} = -2x^2 + 18x - 16$$
$$\cancel{-12} \qquad\qquad -12$$

$$-2x^2 + 18x - 28 = 0$$

$$-2(x^2 - 9x + 14) = 0$$

$$-2(x-2)(x-7) = 0$$

If we sell 2000 or 7000 units we will earn $12,000

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

j. The equation of $P^{-1}(x)$. $a(x-h)^2+k$

$$P(x) = -2(x-4.5)^2 + 24.5$$

$$y = -2(x-4.5)^2 + 24.5$$

$$x = -2(y-4.5)^2 + \cancel{24.5}$$

$$-24.5 \qquad -24.5$$

$$\frac{x-24.5}{-2} = \frac{\cancel{-2}(y-4.5)^2}{\cancel{-2}}$$

$$\sqrt{-\frac{1}{2}(x-24.5)} = \sqrt{(y-4.5)^2}$$

$$y = \sqrt{-\frac{1}{2}(x-24.5)} + 4.5$$

Action!

Profit Functions

Given the demand function, $p(x) = -2x + 20$ and the cost function, $C(x) = 2x + 16$, in thousands of dollars, determine:

k. The domain and range of $\boxed{P(x)}$ and $\boxed{P^{-1}(x)}$.

domain

$\{x \in \mathbb{R} \mid x \geq 0\}$

Range $\{P(x) \in \mathbb{R} \mid P(x) \leq 24.5\}$

domain $\{x \in \mathbb{R} \mid x \leq 24.5\}$

range $\{P^{-1}(x) \in \mathbb{R} \mid P^{-1}(x) \geq 0\}$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}\left(x + 3\sqrt{3}\right)\left(x - 5\sqrt{3}\right)$,

determine, using exact values,

a. The zeros of the function.

$-3\sqrt{3}$ and $5\sqrt{3}$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(x + 3\sqrt{3})(x - 5\sqrt{3})$,

determine, using exact values,

b. The coordinates of the vertex.

$-3\sqrt{3}$ and $5\sqrt{3}$

x-value

$$\frac{-3\sqrt{3} + 5\sqrt{3}}{2}$$

$$\frac{2\sqrt{3}}{2}$$

$$\sqrt{3}$$

<u>y-value</u>

$$y = -2\sqrt{6}\,(\sqrt{3}+3\sqrt{3})(\sqrt{3}-5\sqrt{3})$$
$$= -2\sqrt{6}\,(4\sqrt{3})(-4\sqrt{3})$$
$$= -2\sqrt{6}\,(-16\cdot 3)$$
$$= -2\sqrt{6}\,(-48)$$
$$= 96\sqrt{6}$$

$\therefore$ vertex is $(\sqrt{3},\ 96\sqrt{6})$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(x + 3\sqrt{3})(x - 5\sqrt{3})$,

determine, using exact values,

c. The value of the y-intercept.

$(-2\sqrt{6})(3\sqrt{3})(-5\sqrt{3})$

$= 30\sqrt{54}$

$= 30\sqrt{9 \times 6}$

$= 90\sqrt{6}$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(x + 3\sqrt{3})(x - 5\sqrt{3})$,

determine, using exact values,

d. The vertex form equation of the parabola.

$f(x) = -2\sqrt{6}(x - \sqrt{3})^2 + 96\sqrt{6}$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(x + 3\sqrt{3})(x - 5\sqrt{3})$,

determine, using exact values,

e. The standard form equation of the paraboa.

$f(x) = -2\sqrt{6}\left(x^2 - 5\sqrt{3}x + 3\sqrt{3}x - 15(3)\right)$

$f(x) = -2\sqrt{6}\left(x^2 - 2\sqrt{3}x - 45\right)$

$f(x) =$

$= -2\sqrt{6}x^2 + 4\sqrt{18}x + 90\sqrt{6}$

$= -2\sqrt{6}x^2 + 12\sqrt{2}x + 90\sqrt{6}$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(\cancel{x}4 + 3\sqrt{3})(\cancel{x}4 - 5\sqrt{3})$,

determine, using exact values,

f. The value of $f(x)$ when $x = 4$.

$$f(4) = -2\sqrt{6}(4 + 3\sqrt{3})(4 - 5\sqrt{3})$$

$$= -2\sqrt{6}(16 - 20\sqrt{3} + 12\sqrt{3} - 15(3))$$

$$= -2\sqrt{6}(16 - 8\sqrt{3} - 45)$$

$$= -32\sqrt{6} + 16\sqrt{18} + 90\sqrt{6}$$

$$= 58\sqrt{6} + 16\sqrt{9 \times 2}$$

$$= 58\sqrt{6} + 48\sqrt{2}$$

Action!

Get Radical

Given $f(x) = -2\sqrt{6}(x + 3\sqrt{3})(x - 5\sqrt{3})$,

determine, using exact values,

g. The value of $f(x)$ when $x = 4\sqrt{3}$.

$f(4\sqrt{3}) = -2\sqrt{6}(4\sqrt{3} + 3\sqrt{3})(4\sqrt{3} - 5\sqrt{3})$

$= -2\sqrt{6}(7\sqrt{3})(-1\sqrt{3})$

$= (-14\sqrt{18})(-1\sqrt{3})$

$= 14\sqrt{54}$

$= 14\sqrt{9 \times 6}$

$= 42\sqrt{6}$