

What's Going On?

Checking In

Weekend Assignment

Minds on

What's my Inverse?

Action!

Determining the Inverse of a
Quadratic Function

Consolidation

You Try!

Learning Goal - I will be able to determine the inverse of a quadratic function given its graph or equation.

Checking In

LGL

The profit, $P(a)$, of a particular company, in thousands of dollars, is modelled by the function

$$P(a) = -5a^2 + 400a - 2550$$

where a is the amount spent on advertising, in thousands of dollars.

What is the maximum profit possible?

Completing the Square

$$\begin{aligned} P(a) &= -5[a^2 - 80a + 1600 - 1600] - 2550 \\ &= -5[(a - 40)^2] + \overset{(-5)(-1600)}{8000} - 2550 \\ &= -5(a - 40)^2 + 5450 \end{aligned}$$

Max profit is \$5,450,000

Checking In

LGL

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Factoring

$$P(a) = -5(a^2 - 80a + 510)$$

I can't do it!

Checking In

LGL

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What is the maximum profit possible?

Symmetrical Points

$$P(a) = -5a(a - 80) - 2550$$

axis of symmetry at 40

$$\begin{aligned} P(40) &= -5(40)^2 + 400(40) - 2550 \\ &= -5(1600) + 16000 - 2550 \\ &= 5,450 \end{aligned}$$

Max profit is, AGAIN, \$5,450,000

Checking In

LGL

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where a is the amount spent on advertising, in thousands of dollars.

What is the maximum profit possible?

Quadratic Formula

$$a = \frac{-400 \pm \sqrt{400^2 - 4(-5)(-2550)}}{2(-5)}$$

$$= \frac{-400 \pm \sqrt{160,000 - 51,000}}{-10}$$

$$= \frac{-400 \pm 343.57}{-10}$$

$$= \frac{-400 + 343.57}{-10} \quad \text{or} \quad \frac{-400 - 343.57}{-10}$$

$$= 0.643 \quad \text{or} \quad 79.357$$

$$\begin{aligned} \text{axis of symmetry} &= \frac{0.643 + 79.357}{2} \\ &= \frac{80.000}{2} \\ &= 40 \end{aligned}$$

$$\begin{aligned} P(40) &= -5(40)^2 + 400(40) - 2550 \\ &= 5,450 \end{aligned}$$

Checking In

LGL

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What is the maximum profit possible?

Please do this before you RAFT

Checking In

Profit Models

Wrapping up Maximum and Minimum Values

Profit Models

Quadratics are commonly used as profit models.

The basic idea behind this is that when you first start a business, as your sales increase your profits increase at a rapid rate.

However, you eventually hit a point where you can no longer meet demand. At this point you need to hire more employees, invest in real estate or other capital projects. When you get here, your profits decrease.

Profit Models

When we discuss profit models we have

A Cost Function: $C(x)$

A Demand Function: $p(x)$

The number of units sold: (x)

The Revenue Function: $R(x) [p(x)](x)$

A Profit Function: $P(x)$ *Revenue* – *Cost*
 $[p(x)](x) – C(x)$

Profit Models

Example

The demand function for a new magazine is $p(x) = -6x + 40$, where $p(x)$ represents the selling price, in thousands of dollars, of the magazine and x is the number sold, in thousands.

The cost function is $C(x) = 4x + 48$. Calculate the maximum profit and the number of magazines sold that will produce the maximum profit.

$$\begin{aligned}
 P(x) &= R(x) - C(x) \\
 &= (x)(-6x + 40) - (4x + 48) \\
 &= -6x^2 + 40x - 4x - 48
 \end{aligned}$$

$$= -6x^2 + 36x - 48$$

$$P(x) = -6x(x - 6) - 48$$

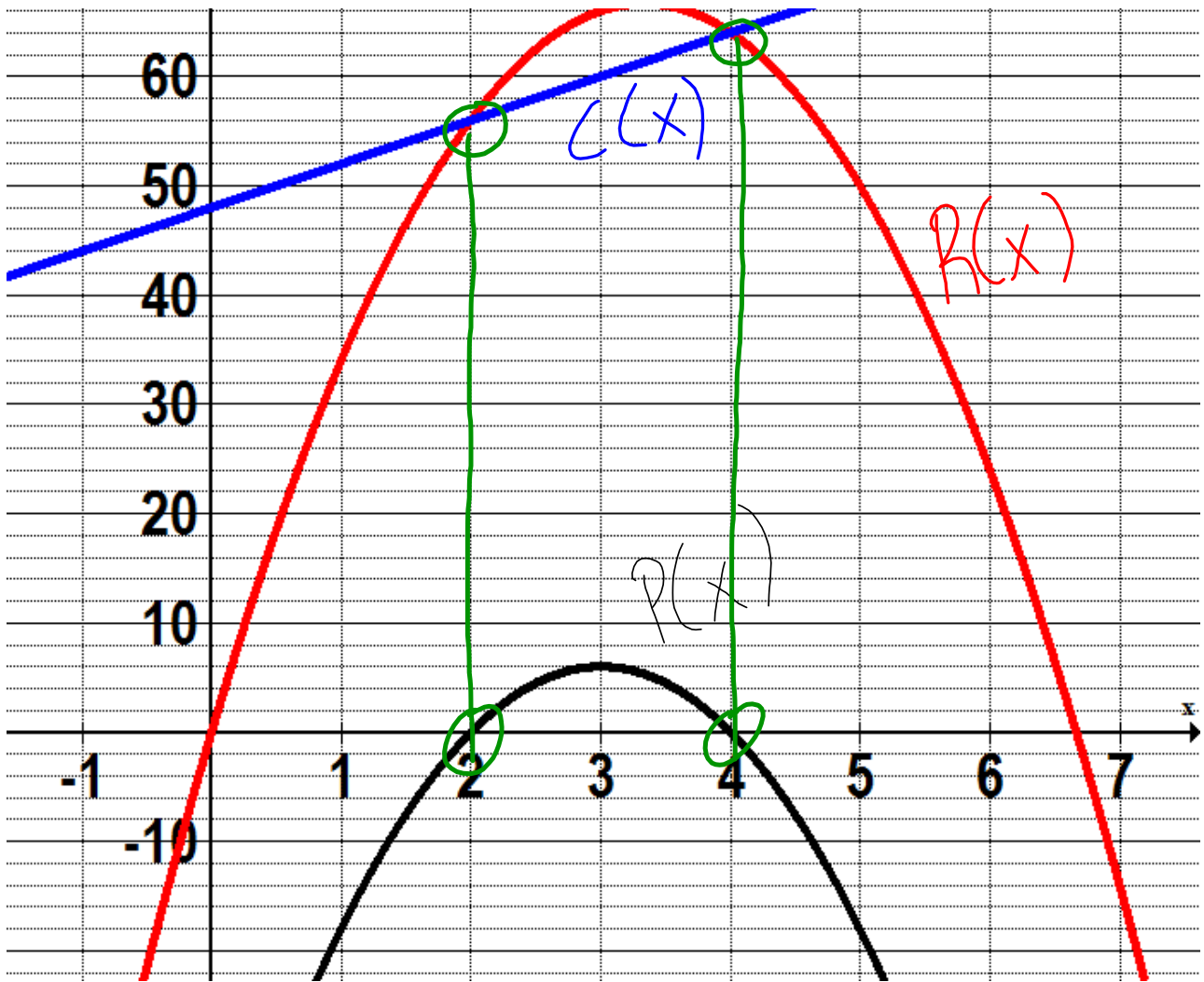
axis of symmetry when $x = 3$

$$P(3) = -6(3)^2 + 36(3) - 48$$

$$= -54 + 108 - 48$$

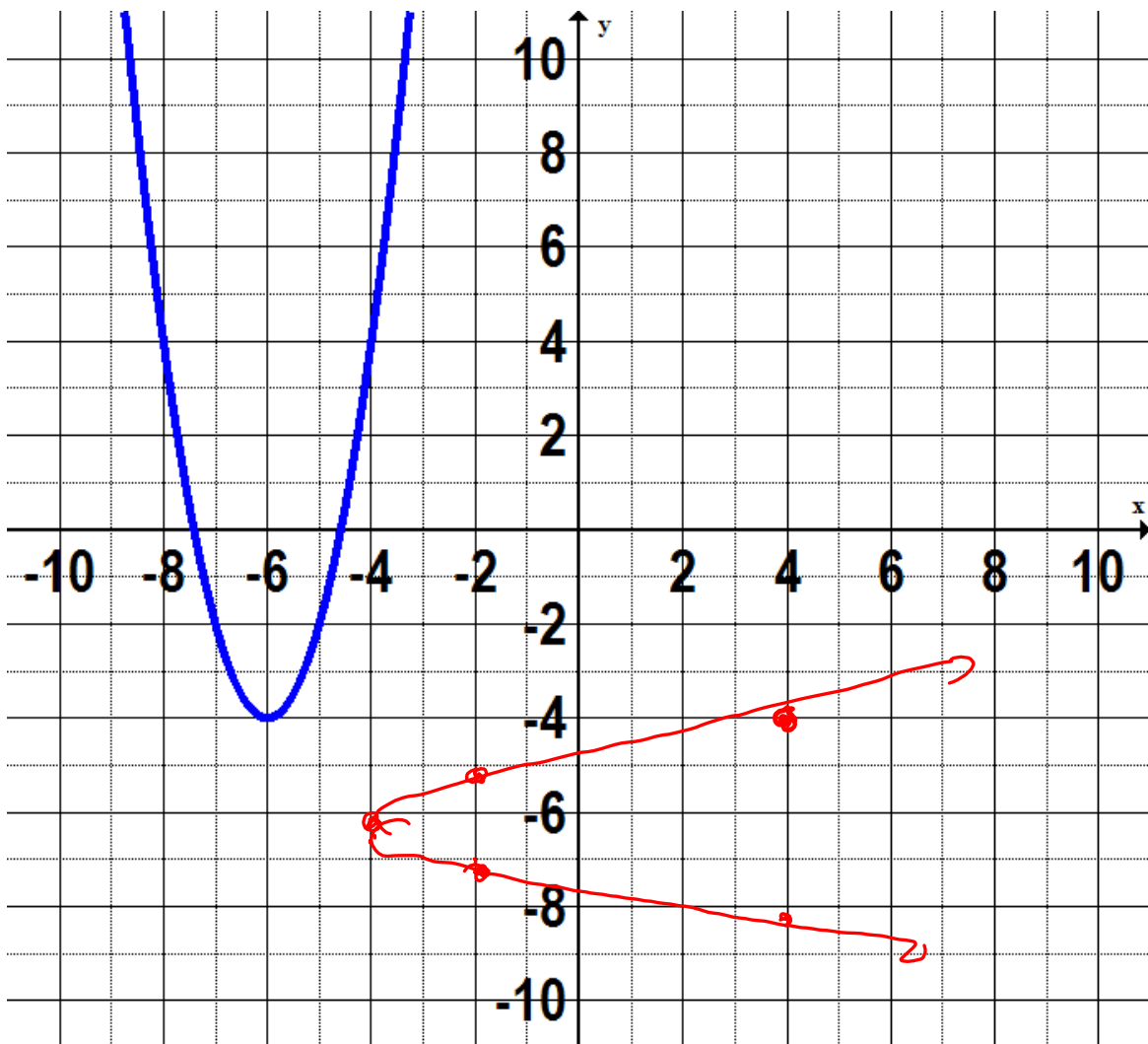
$$= 6$$

\therefore max profit is \$6,000



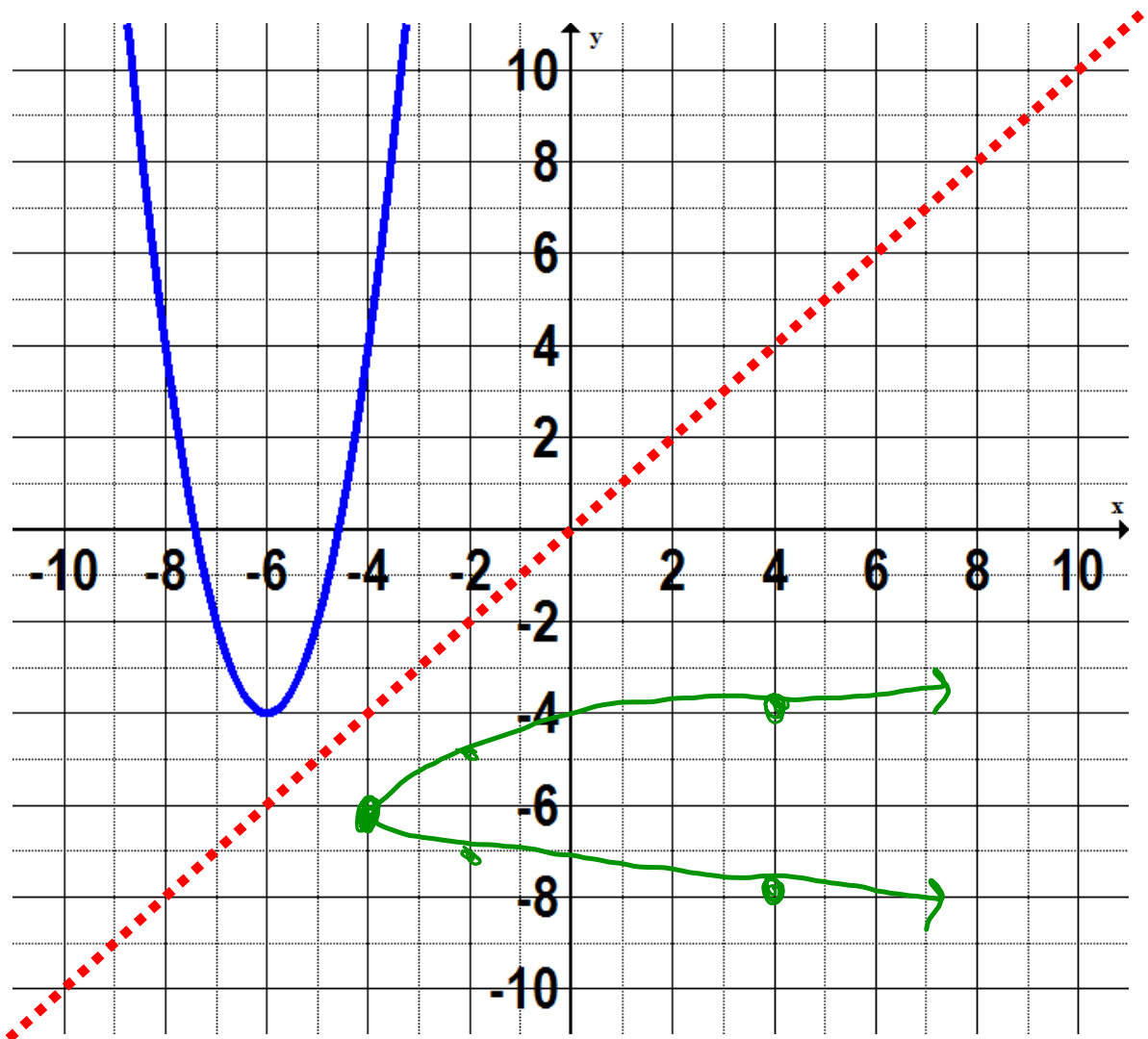
Minds on

Sketch my Inverse



Minds on

Sketch my Inverse



Action!

What Does the Inverse Mean?

The inverse of a function amounts to switching the direction of a relationship.

Specifically, the independent variable and dependent variable get switched.

Action!

Determining the Inverse of a Quadratic Function

As we saw in Unit 1, to determine the inverse of the graph of a function:

Reflect the points about the line $x = y$ or
 $y = x$

Action!

Determining the Inverse of a Quadratic Function

As we saw in Unit 1, to determine the inverse of the **table of values** of a function:

Switch the columns!

Action!

Determining the Inverse of a Quadratic Function

As we saw in Unit 1, to determine the inverse of the equation of a function:

1. Switch the variables (often x and y)
2. Solve for the new y

Action!

What Does the Inverse Mean?

The area of a square is given by the equation

$$A = s^2$$

where s is the length of a side

The side length of a square is given by the equation

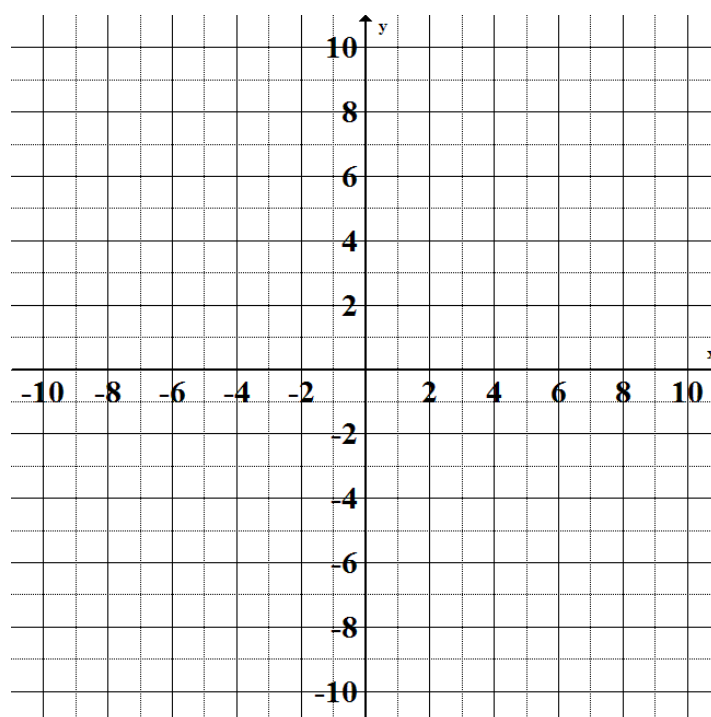
$$s = \sqrt{A}$$

Consolidation

You Try!

Example 1: Given the quadratic function $f(x) = -3(x + 7)^2 + 8$

- Graph $f(x)$ and its inverse
- Determine whether the inverse of $f(x)$ is also a function
- Determine the equation of the inverse
- State the domain and range of $f(x)$ and the inverse relation



Consolidation

Homework