

Determining the Inverse of a Quadratic Function

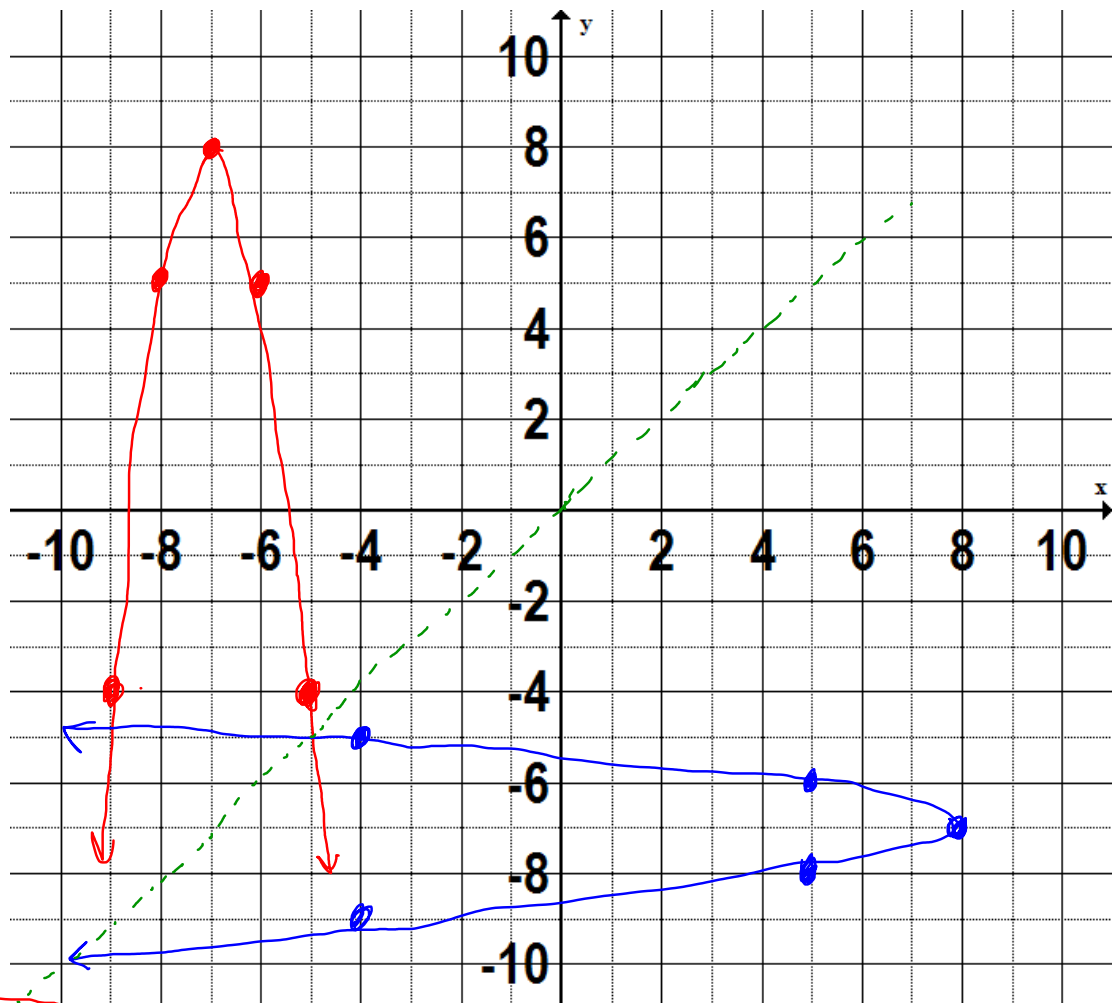
Example

Given the quadratic function $f(x) = -3(x + 7)^2 + 8$

- Graph $f(x)$ and its inverse
- Determine whether the inverse of $f(x)$ is also a function NO, fails vertical line test
- Determine the equation of the inverse
- State the domain and range of $f(x)$ and the inverse relation

$$\begin{aligned}
 c. \quad y &= -3(x+7)^2 + 8 \\
 x &= -3(y+7)^2 + 8 \\
 -8 & \qquad \qquad -8 \\
 \underline{x-8} &= \underline{-3(y+7)^2} \\
 -3 & \qquad \qquad -3 \\
 -\frac{1}{3}(x-8) &= (y+7)^2
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{-\frac{1}{3}(x-8)} &= \sqrt{(y+7)^2} \\
 y+7 &= \sqrt{-\frac{1}{3}(x-8)} \\
 -7 & \qquad \qquad -7 \\
 \boxed{y} &= \boxed{\sqrt{-\frac{1}{3}(x-8)} - 7}
 \end{aligned}$$



$$f(x): \text{domain} = \{x \in \mathbb{R}\} \quad \text{range} = \{f(x) \in \mathbb{R} \mid f(x) \leq 8\}$$

$$f^{-1}(x): \text{domain} = \{x \in \mathbb{R} \mid x \leq 8\} \\
 \text{range} = \{f^{-1}(x) \in \mathbb{R}\}$$