

Solutions

MCR3U – Unit 2 Review: The Big Questions

1. Simplify and state restrictions. Be sure to identify asymptotes and holes.

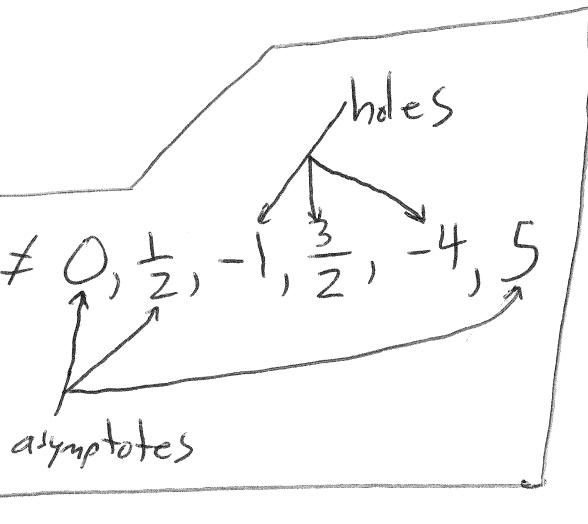
$$f(x) = \frac{-2x^2 + 2}{x^2 - x - 20} \div \frac{2x^2 - x - 3}{x^2 + 8x + 16} \times \frac{6x^2 - 7x - 3}{12x^2 - 6x}$$

$$\begin{aligned}
 &= \frac{-2(x^2 - 1)}{(x-5)(x+4)} \times \frac{(x+4)(x+4)}{2x^2 + 2x - 3x - 3} \times \frac{6x^2 - 9x + 2x - 3}{6x(2x-1)} \\
 &= \frac{-2(x+1)(x-1)}{(x-5)(x+4)} \times \frac{(x+4)(x+4)}{2x(x+1) - 3(x+1)} \times \frac{3x(2x-3) + 1(2x-3)}{6x(2x-1)} \\
 &= \frac{-2(x+1)(x-1)}{(x-5)(x+4)} \times \frac{(x+4)(x+4)}{(x+1)(2x-3)} \times \frac{(2x-3)(3x+1)}{6x(2x-1)} \\
 &\quad \underbrace{x \neq 5, -4, -1, \frac{3}{2}, 0, \frac{1}{2}}_{\text{holes}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-2(x+1)(x-1)(x+4)(x+4)(2x-3)(3x+1)}{6x(x-5)(x+4)(x+1)(2x-3)(2x-1)}
 \end{aligned}$$

$$= \frac{-2(x-1)(x+4)(3x+1)}{6x(x-5)(2x-1)}$$

$$= \frac{-1(x-1)(x+4)(3x+1)}{3x(x-5)(2x-1)} ; x \neq 0, \frac{1}{2}, -1, \frac{3}{2}, -4, 5$$



2. Given $f(x)$ above and $g(x)$ below,
- Is $g(x)$ equivalent to $f(x)$? Explain and show your work.
Hint: you cannot factor $g(x)$!
 - Strengthen/verify your claim by substituting two x -values into $f(x)$ and $g(x)$.

$$g(x) = \frac{-3x^3 - 10x^2 + 9x + 4}{6x^3 - 33x^2 + 15x}$$

d. I'll expand $f(x)$ from #1

$$\begin{aligned} f(x) &= \frac{-1(\cancel{x-1})(\cancel{x+4})(3x+1)}{\cancel{3x}(\cancel{x-5})(2x-1)} \\ &= \frac{(-x+1)(3x^2+x+12x+4)}{(3x^2-15x)\cancel{(2x-1)}} \\ &= \frac{-x(3x^2+13x+4) + 1(3x^2+13x+4)}{6x^3-3x^2-30x^2+15x} \\ &= \frac{-3x^3-13x^2-4x+3x^2+13x+4}{6x^3-33x^2+15x} \\ &= \frac{-3x^3-10x^2+9x+4}{6x^3-33x^2+15x} \quad * \text{same as } g(x) \\ &\boxed{\therefore f(x) = g(x)} \end{aligned}$$

b. ~~$f(1)$~~ *Plug into original $f(x)$ and $g(x)$

$$\begin{aligned} f(1) &= \frac{-2(1)^2+2}{(1)^2-(1)-20} \div \frac{2(1)^2-(1)-3}{(1)^2+8(1)+16} \times \frac{6(1)^2-7(1)-3}{12(1)^2-6(1)} \\ &= \frac{-2+2}{-20} \div \frac{-2}{26} \times \frac{-4}{6} \\ &= 0 \times -3 \times -\frac{2}{3} \end{aligned}$$

$$\boxed{f(1) = 0}$$

✓ same

$$\begin{aligned} g(1) &= \frac{-3(1)^3-10(1)^2+9(1)+4}{6(1)^3-33(1)^2+15(1)} \\ &= \frac{-3-10+9+4}{6-33+15} \\ &= \frac{0}{-12} \\ &= \boxed{0} \end{aligned}$$

3. Given the rational expression of $h(x)$ below,
- Simplify and state restrictions.
 - Is $h(x)$ equivalent to $f(x)$ or $g(x)$?
 - Strengthen/verify your claim by substituting the same 2 x -values into $h(x)$ that you substituted into $f(x)$ and $g(x)$ in question 2b.

$$h(x) = \frac{x^2 + 3x - 4}{3x^2 - 15x} - \frac{-6x - 2}{4x^2 - 22x + 10}$$

$$\begin{aligned}
a. \quad h(x) &= \frac{(x+4)(x-1)}{3x(x-5)} - \frac{-2(3x+1)}{2(2x^2-11x+5)} \\
&= \frac{(x+4)(x-1)}{3x(x-5)} - \frac{-2(3x+1)}{2(2x^2-x-10x+5)} \\
&= \frac{(x+4)(x-1)}{3x(x-5)} - \frac{-2(3x+1)}{2(x(2x-1)-5(2x-1))} \\
&= \frac{(x+4)(x-1)}{3x(x-5)} - \frac{-2(3x+1)}{2(x-5)(2x-1)} \\
&\quad \text{---} \\
&= \frac{2(2x-1)}{2(2x-1)} \cdot \frac{(x+4)(x-1)}{3x(x-5)} - \frac{3x(-2)(3x+1)}{3x \cdot 2(x-5)(2x-1)} \\
&= \frac{\cancel{2}(2x-1)(x+4)(x-1) - (3x)\cancel{(-2)}(3x+1)}{(2)(3x)(x-5)(2x-1)} \\
&= \frac{(4x-2)(x^2+3x-4) - (-6x)(3x+1)}{(6x)(2x^2-11x+5)(x-5)(2x-1)} \\
&= \frac{4x^3 + 12x^2 - 16x - 2x^2 - 6x + 8 + 18x^2 + 6x}{(6x)(x-5)(2x-1)} \\
&= \frac{4x^3 + 28x^2 - 16x + 8}{(6x)(x-5)(2x-1)}, \quad x \neq 0, 5, \frac{1}{2}
\end{aligned}$$

b. /not the same!
because when simplified,
the expressions are
not equivalent

$$\begin{aligned}
h(1) &= \frac{(1)^2 + 3(1) - 4}{3(1)^2 - 15(1)} - \frac{-6(1) - 2}{4(1)^2 - 22(1) + 10} \\
&= \frac{0}{-12} - \frac{-8}{-8} \\
&= 0 - 1 \\
&= -1
\end{aligned}$$

$$\begin{aligned}
&h(1) = f(1) = g(1) = 0 \\
&\therefore h(1) \neq f(1) \text{ and } h(1) \neq g(1)
\end{aligned}$$