What's Going On?

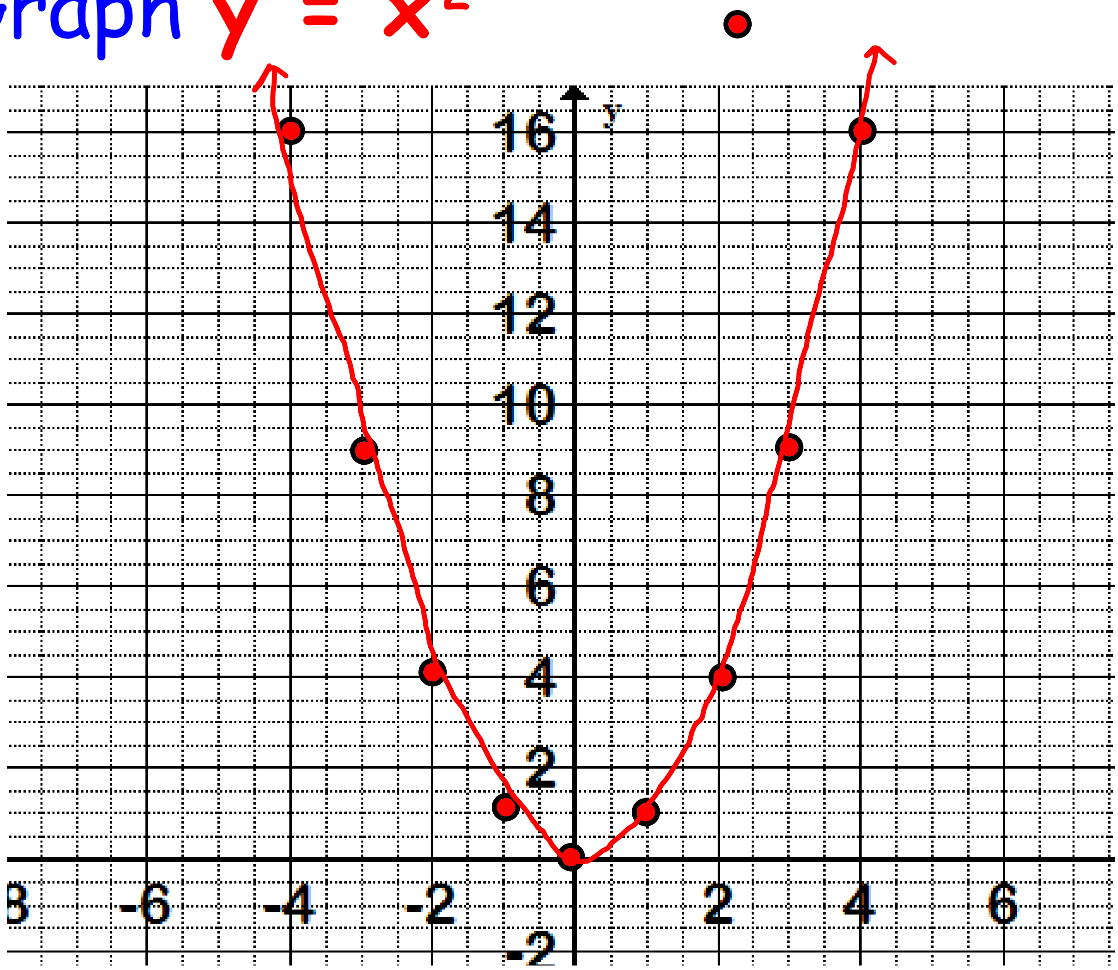
Checking In	
Minds on	Back to Basics
Action!	Factoring & Completing the Square
Consolidation	Self-Test

Learning Goal - I will be able to graph, factor and problem solve with quadratics.

Minds on

Back to Basics

Graph $y = x^2$



This will be our base case.

Every other function that we graph will be done by a series of transformations on $y = x^2$.

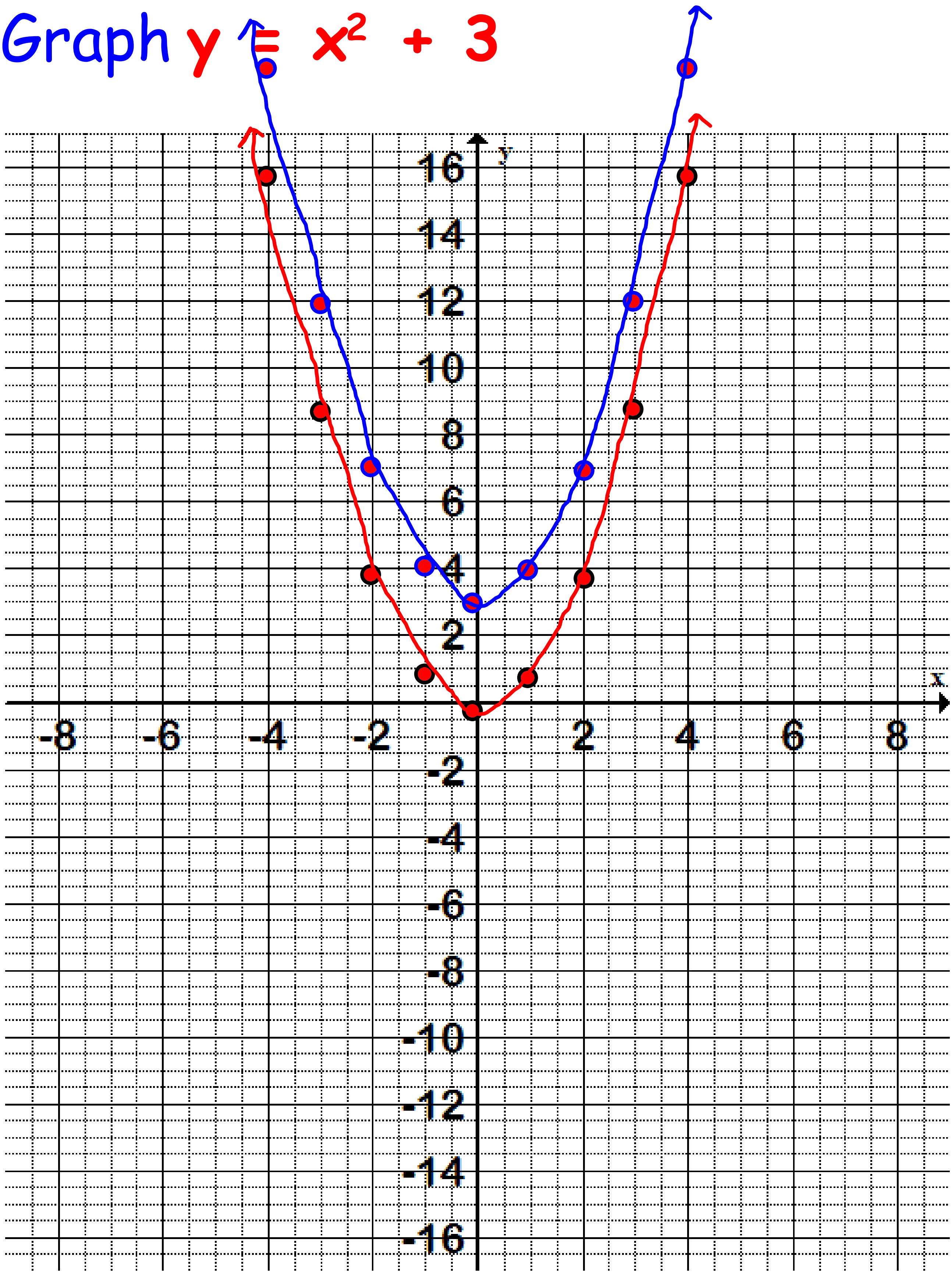
To graph $y = x^2$ we start with the vertex (**0**, **0**) and use the step pattern:

over 1, **up 1**
over 1, **up 3**
over 1, **up 5**
over 1, **up 7**...

Remember that as we move on to different functions, the only things that change are the coordinates of the vertex and the vertical component of the step pattern.

Minds on

Graph $y = x^2 + 3$



To graph $y = x^2 + 3$, we need to apply

a **vertical shift** to $y = x^2$ **up 3 units**.

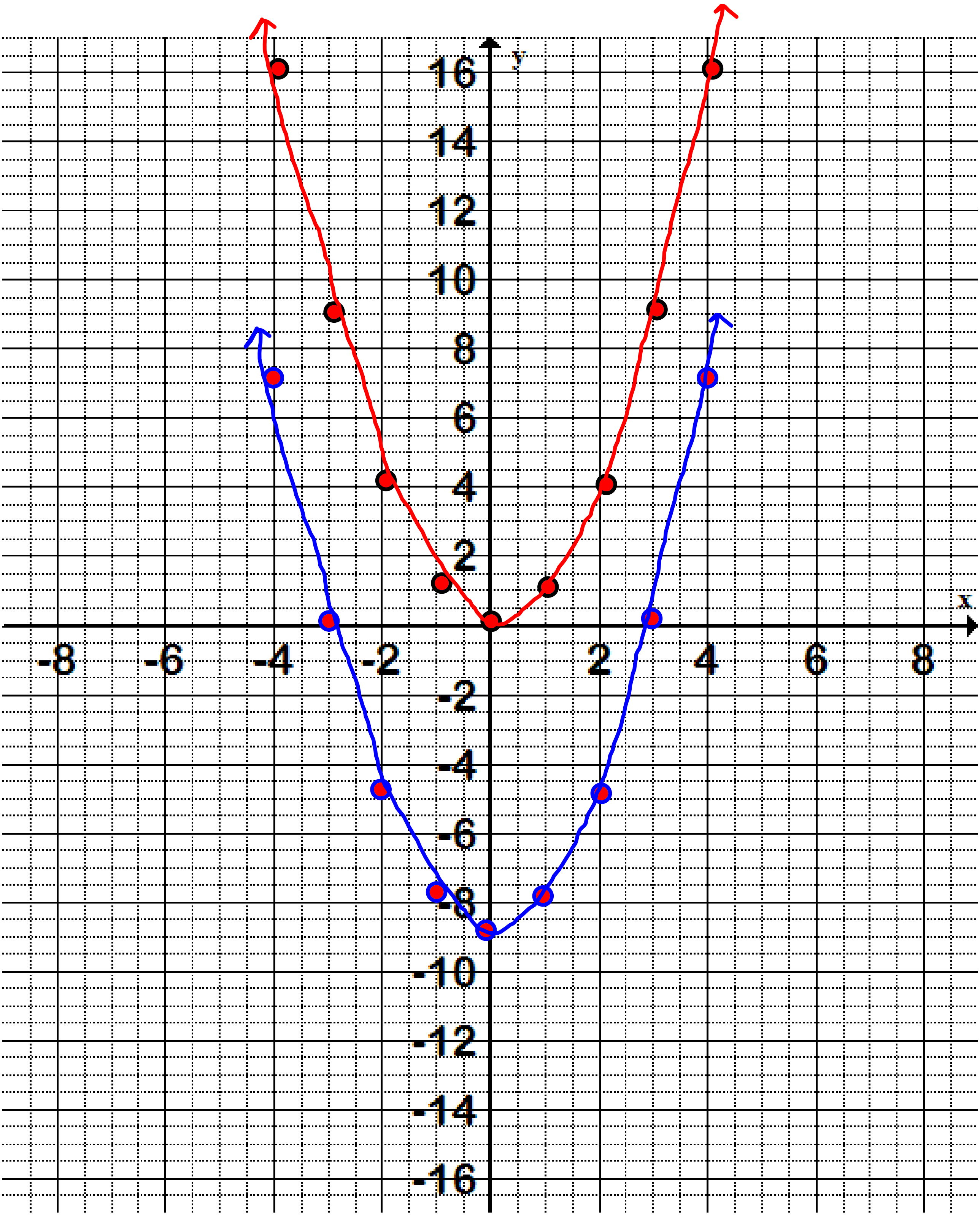
The vertex is (0,**3**).

we use the step pattern:

over 1, **up 1**
over 1, **up 3**
over 1, **up 5**
over 1, **up 7**...

Minds on

Graph $y = x^2 - 9$



To graph $y = x^2 - 9$, we need to apply

a **vertical shift** to $y = x^2$ **down 9 units**.

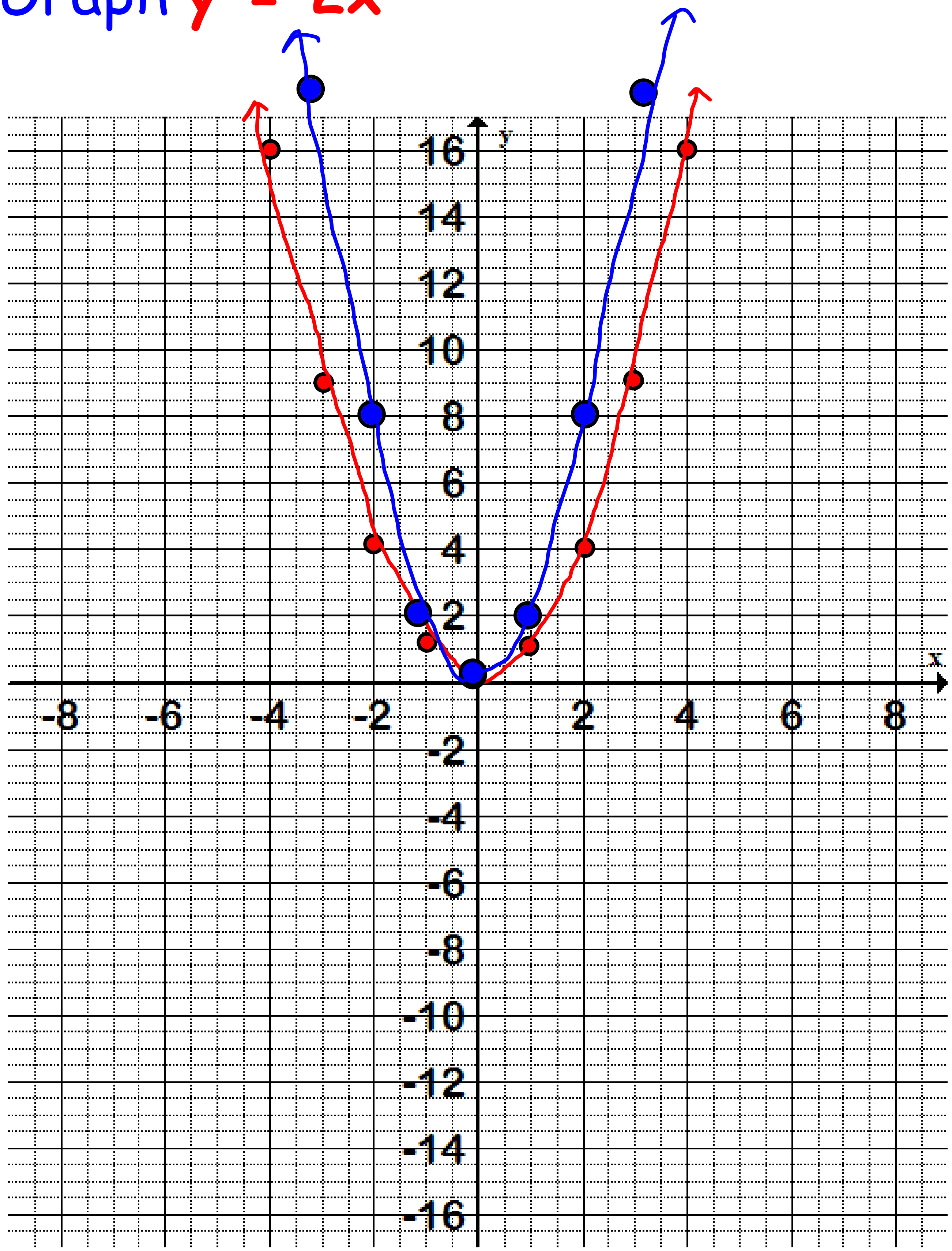
The vertex is (0, **-9**).

we use the step pattern:

over 1, **up 1**
over 1, **up 3**
over 1, **up 5**
over 1, **up 7**...

Minds on

Graph $y = 2x^2$



To graph **$y = 2x^2$**, we need to apply

a **vertical stretch to $y = x^2$ of 2**.

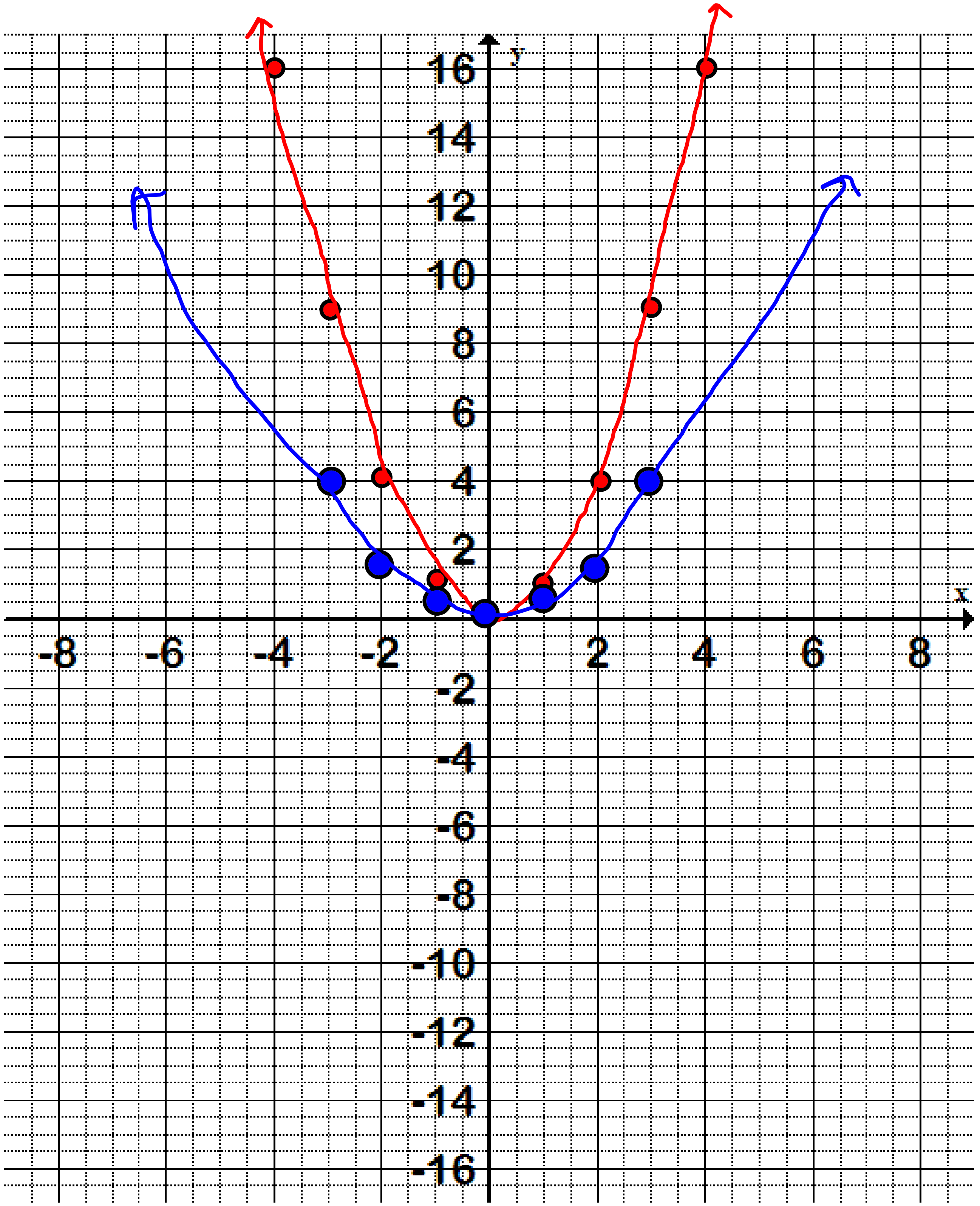
The vertex is (0, 0).

we use the step pattern:

over 1, **up 2**
over 1, **up 6**
over 1, **up 10**
over 1, **up 14**...

Minds on

Graph $y = 0.5x^2$



To graph **$y = 0.5x^2$**, we need to apply

a **vertical compression** **to $y = x^2$**

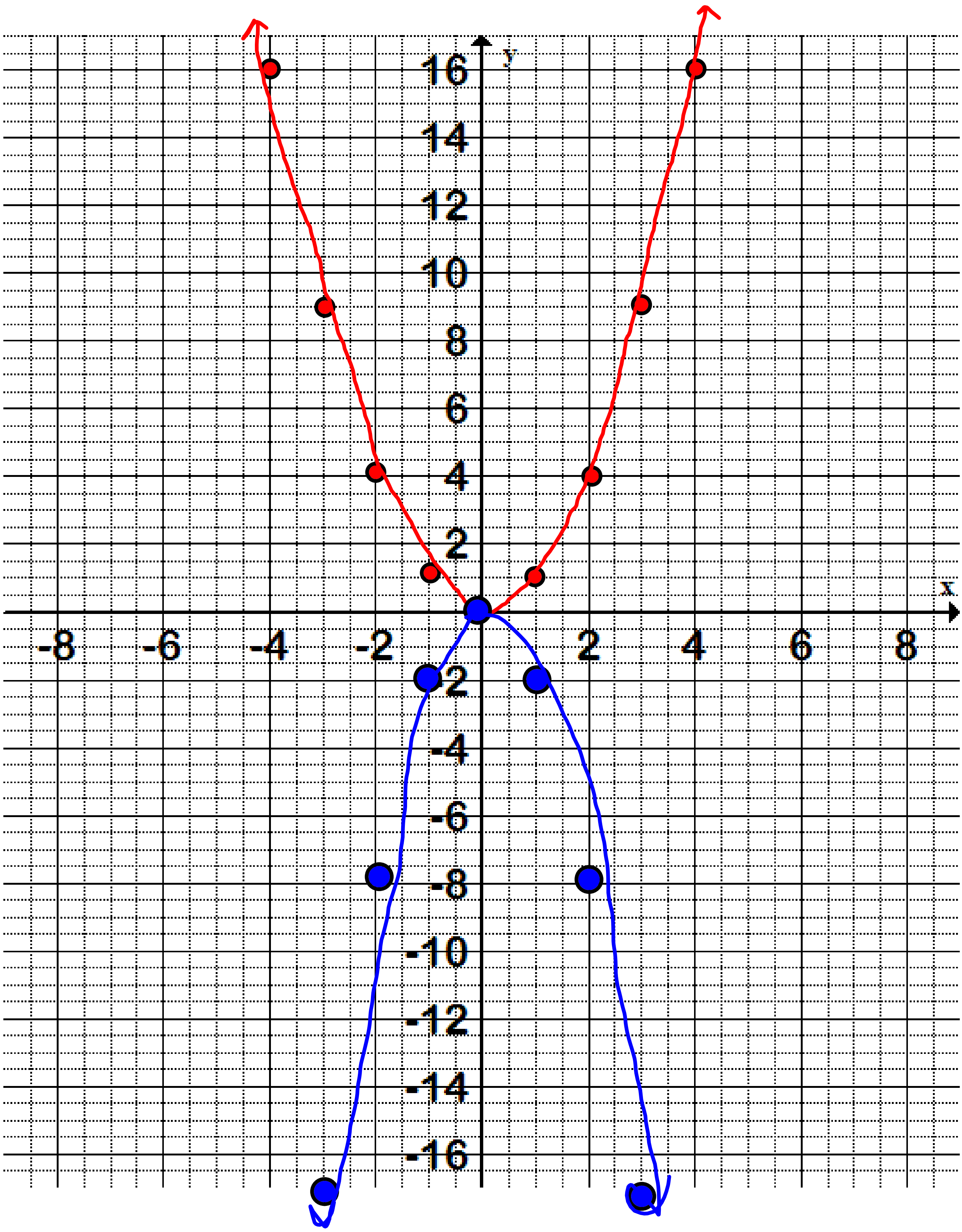
of 0.5, the vertex is (0, 0).

we use the step pattern:

over 1, **up 0.5**
over 1, **up 1.5**
over 1, **up 2.5**
over 1, **up 3.5**...

Minds on

Graph $y = -2x^2$



To graph **$y = -2x^2$**, we need to apply

a **vertical stretch to $y = x^2$ of 2,** and

a **reflection in the x-axis.**

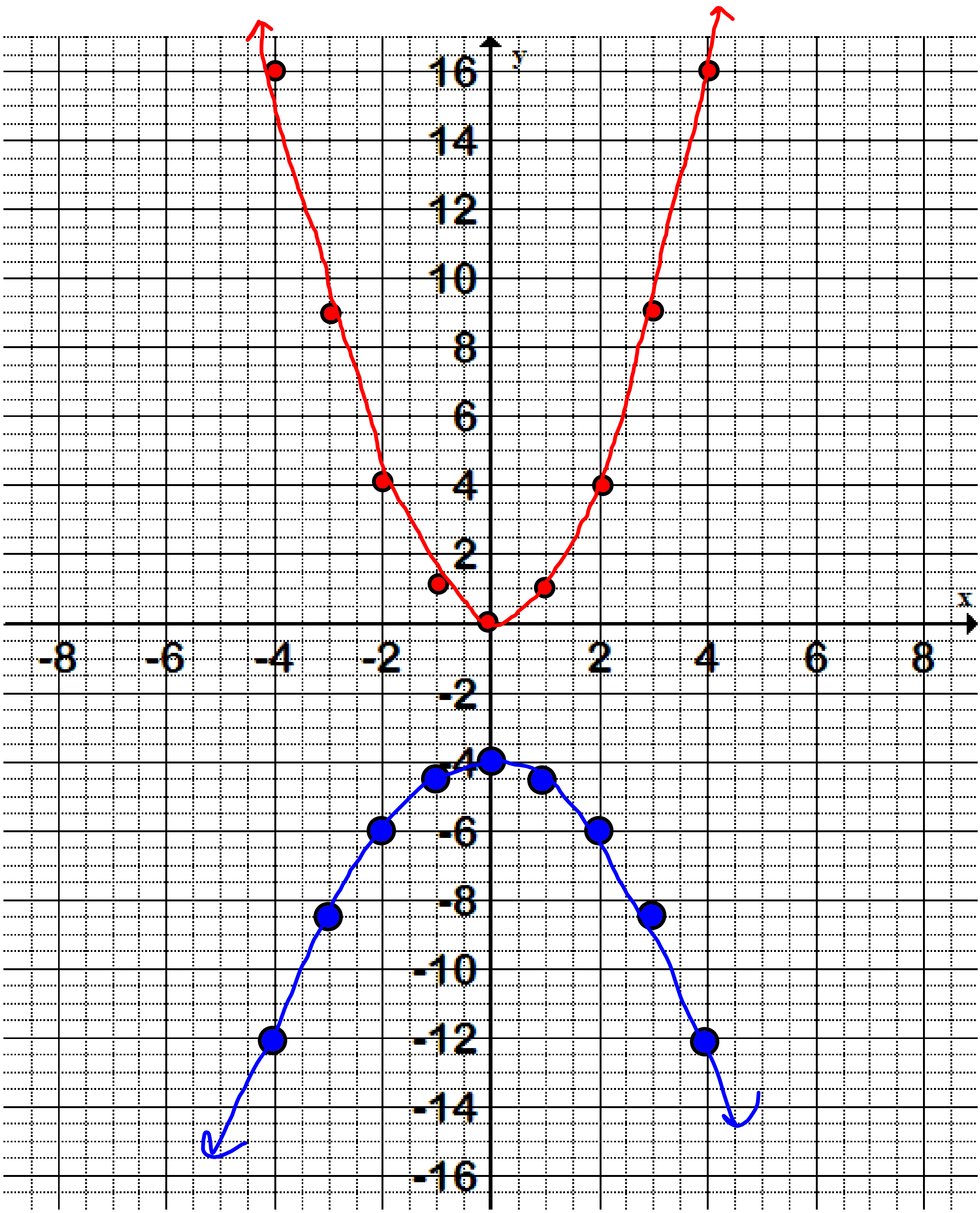
The vertex is (0, 0).

we use the step pattern:

over 1, **down 2**
over 1, **down 6**
over 1, **down 10**
over 1, **down 14**...

Minds on

Graph $y = -0.5x^2 - 4$



To graph $y = -0.5x^2 - 4$, we need to apply a **vertical compression** **to $y = x^2$** **of 0.5**, a **reflection in the x-axis,** and a **vertical shift** **to $y = x^2$** **down 4 units**

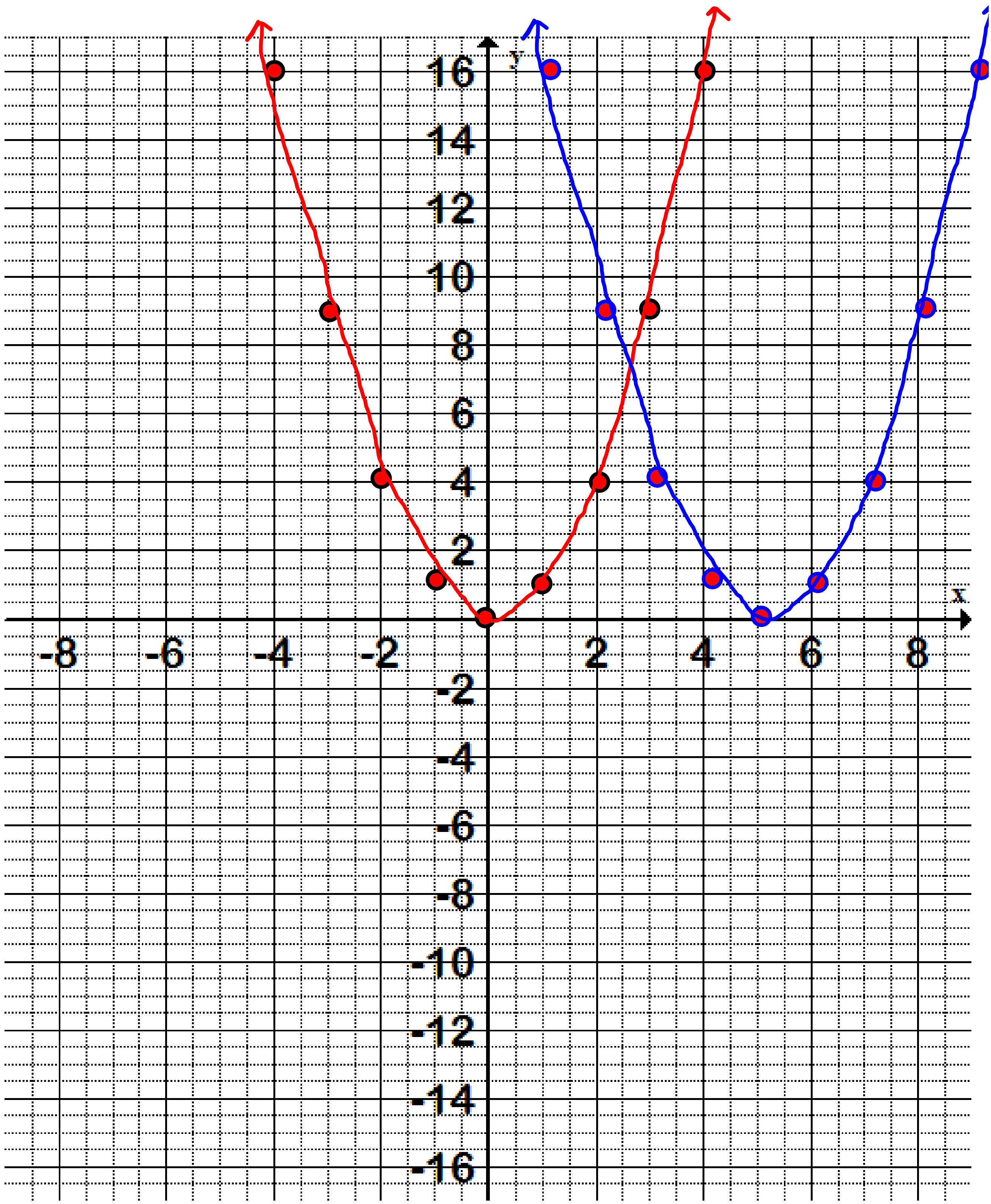
The vertex is (0, **-4**).

we use the step pattern:

over 1, **down 0.5**
over 1, **down 1.5**
over 1, **down 2.5**
over 1, **down 3.5**...

Minds on

Graph $y = (x - 5)^2$



To graph **$y = (x - 5)^2$**, we need to

apply a **horizontal shift** **to $y = x^2$**

right 5 units.

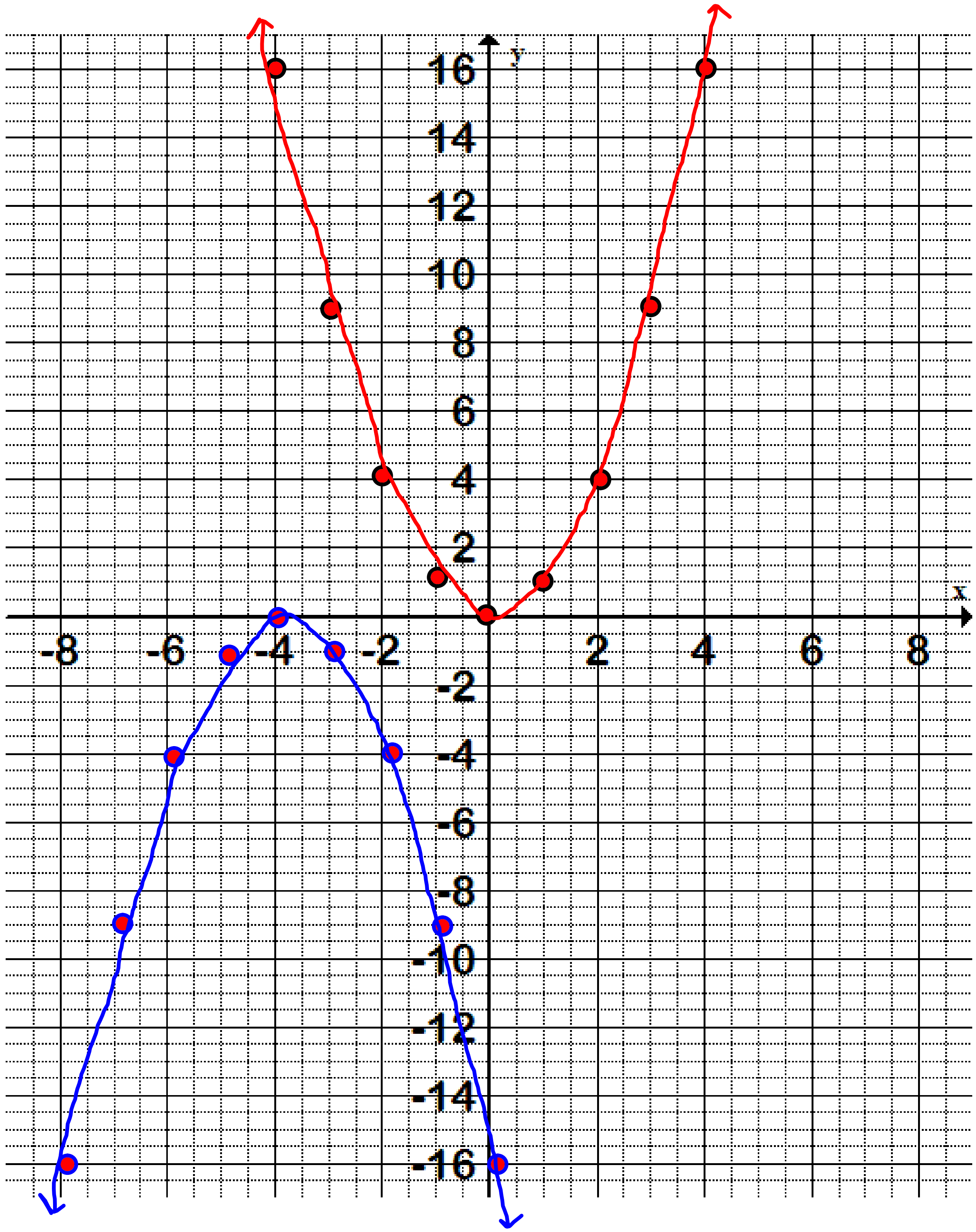
The vertex is (**5**, 0).

we use the step pattern:

over 1, **up 1**
over 1, **up 3**
over 1, **up 5**
over 1, **up 7**...

Minds on

Graph $y = -(x + 4)^2$



To graph **$y = -(x + 4)^2$**, we need to apply

a **horizontal shift** **to $y = x^2$** **left 4 units**,

and a **reflection in the x-axis**.

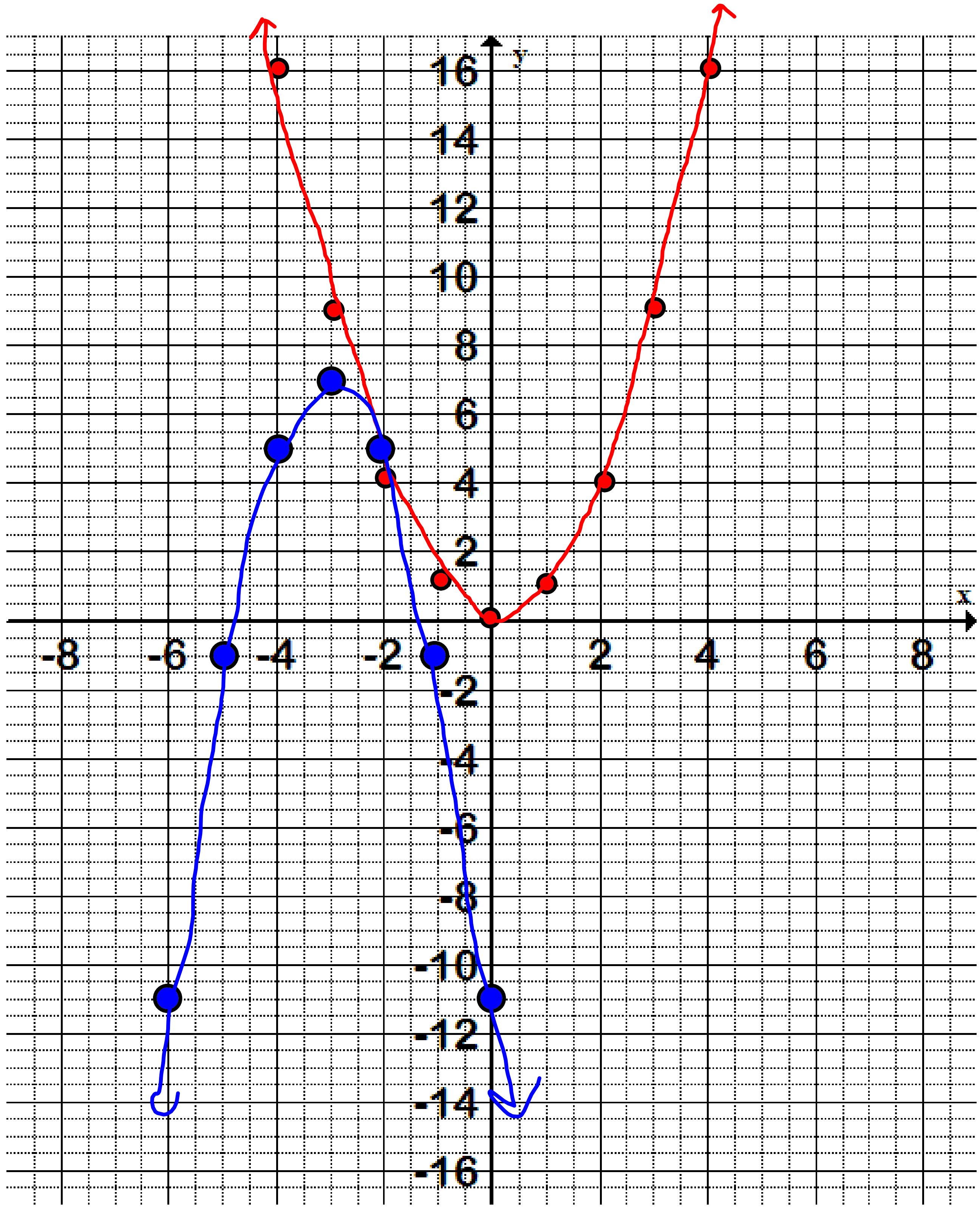
The vertex is (**-4**, 0).

we use the step pattern:

over 1, **down 1**
over 1, **down 3**
over 1, **down 5**
over 1, **down 7**...

Minds on

Graph $y = -2(x + 3)^2 + 7$



Graph $y = -2(x+3)^2 + 7$

y-int?

$$y = -2(x+3)^2 + 7$$

$$y = -2(x+3)(x+3) + 7$$

$$y = -2(x^2 + 3x + 3x + 9) + 7$$

$$y = -2x^2 - 12x - 18 + 7$$

$$y = -2x^2 - 12x - 11$$

To graph $y = -2(x + 3)^2 + 7$, we need to apply a **vertical stretch** to $y = x^2$ **of 2,** a **reflection in the x-axis,** a **vertical shift** to $y = x^2$ **up 7 units** and a **horizontal shift** to $y = x^2$ **left 3 units**

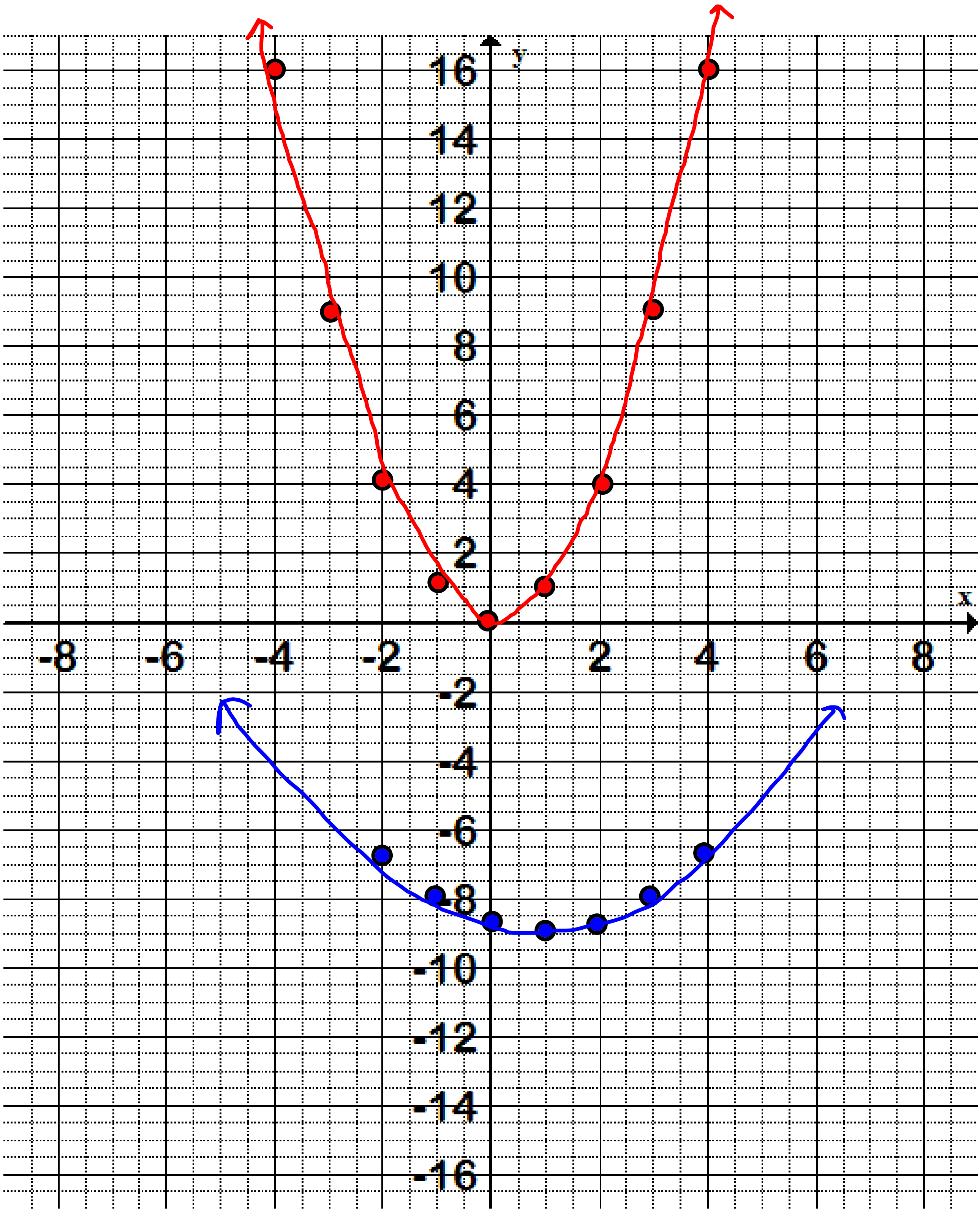
The vertex is (**-3**, **7**).

we use the step pattern:

over 1, **down 2**
over 1, **down 6**
over 1, **down 10**
over 1, **down 14**...

Minds on

Graph $y = 0.25(x - 1)^2 - 9$



To graph **$y = 0.25(x - 1)^2 - 9$**, we need to apply a **vertical compression** **to $y = x^2$ of 0.25,** a **vertical shift** **to $y = x^2$ down 9 units** and a **horizontal shift to $y = x^2$ right 1 unit.**

The vertex is (**1**, **-9**).

we use the step pattern:

over 1, **up 0.25**
over 1, **up 0.75**
over 1, **up 1.25**
over 1, **up 1.75**...

Make it So!

When is the statement below true?

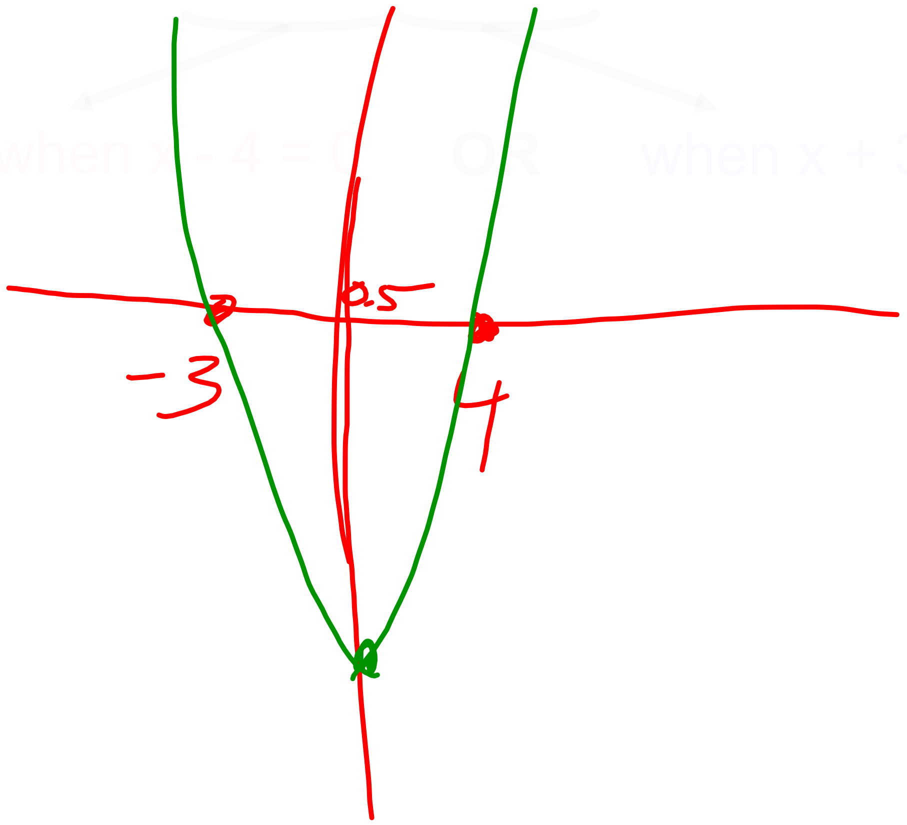
$2(a)(b) = 0$

This equation holds when **either** **a** is 0, **or** when **b** is 0.

Make it So!

When is the statement below true?

$$2(x - 4)(x + 3) = 0$$



Factored Form

A quadratic equation in factored typically looks like:

$$a(x + r)(x + s) = 0$$

where **-r** and **-s** are the **roots** of the equation.

Factored Form

We are often given a standard form equation, and we need to find the factored form by FACTORING!

Factoring is really just "reverse expanding", so let's look at expanding a **basic** quadratic first; this will help us understand what is going on in factoring.

$$(x + r)(x + s) = 0$$

$$= x^2 + sx + rx + rs$$

$$= x^2 + (s+r)x + rs$$

$$x^2 + 5x + 6$$

Solving by Factoring when a = 1!

$ax^2 + bx + c = 0$

$x^2+3x+4x+12$

$x^2 + 7x + 12 = 0$

Find two numbers that **add to b** and **multiply to c**

$(x + 3)(x + 4) = 0$

Therefore, we have roots when:

$(x + 3) = 0$ **AND** $(x + 4) = 0$

$x = -3$

$x = -4$

$x^2 + 8x + 15$

$= (x + 3)(x + 5)$

$m^2 - 25$

$m^2 + 0m - 25$

$= (m - 5)(m + 5)$

$b^2 - 9b - 22$

$(b+2)(b-11)$

Minds on

Factoring when $a \neq 1$!

$$ax^2 + bx + c = 0$$

$$2x^2 - 6x - 20 = 0$$

Common Factor

$$2(x^2 - 3x - 10) = 0$$

Find two numbers that add to -3 and multiply to -10

$$2(x + 2)(x - 5) = 0$$

Therefore, we have roots when:

$(x + 2) = 0$ **AND** $(x - 5) = 0$

$x = -2$ $x = 5$

$-2x^2 + 18x - 40$

$-2(x^2 - 9x + 20)$

$-2(x-4)(x-5)$

Factoring when $a \neq 1$,
and when a is not **a** common factor.

In these cases, the final result looks like this:

$$(Ax + C)(Bx + D) = 0$$

I was going to use this to show you where factoring by grouping comes from, but I won't.

Action!

$6x^2 - 11x - 7$

Solving by Factoring when $a \neq 1$!

$$6x^2 + 13x - 5 = 0$$

Find two numbers that
add to b and multiply to ac

+15 and -2

$$6x^2 + 15x - 2x - 5 = 0$$

$$3x(2x + 5) - 1(2x + 5) = 0$$

$$(3x - 1)(2x + 5) = 0$$

Factor by Grouping

Whoa, what?! So what are the zeros?!

$6x^2 - 11x - 7$

I need to find two numbers that multiply to (6)(-7) and add to -11. Well... (6)(-7) is -42.

So I know I have one positive and one negative number. Because the numbers add to -11 (A NEGATIVE!) I know that I have "more negative" or that the larger of the two numbers is negative.

The numbers are 14 and 3, and the 14 is negative.

$$= 6x^2 + 3x - 14x - 7$$

Now I group the first two terms and the last two terms and common factor each pair.

$$= 3x(2x+1) - 7(2x+1)$$

Finally I common factor each big section (in this case each big section has (2x + 1) in it.

$$= (3x-7)(2x+1)$$

$6x^2 - 11x - 7$

Last time, after the first step, I had:

$$= 6x^2 + 3x - 14x - 7$$

What if I switched the middle two terms?
Would it still work?! Let's see.......

$$= 6x^2 - 14x + 3x - 7$$

$$= 2x(3x - 7) + 1(3x - 7)$$

$$= (3x - 7)(2x + 1)$$

or $(2x + 1)(3x - 7)$

IT WORKED!!

$$(3x - 1)(2x + 5) = 0$$

We can find the roots by setting **either** set of brackets to zero and solving for x.

$$(3x - 1) = 0$$

$$x = +\frac{1}{3}$$

$$(2x + 5) = 0$$

$$x = -\frac{5}{2}$$

So the zeros are 1/3 and -5/2

$$(3x - 1)(2x + 5) = 0$$

But hold on!! That's not in the form:

$$a(x + r)(x + s) = 0$$

It's true :(

Sometimes,

life is complicated.

Special Cases

Solving $ax^2 + bx + c$ by factoring when c = 0.

"Solve" $3x^2 + 6x = 0$

Common Factor

$$3x(x + 2) = 0$$

Hmmmmmmm. so what are the zeros?

Well, I know that this equation will be true when $(x + 2) = 0$, so -2 is one of the zeros. I also know that if I plug in a zero for x, the 3x part will become zero and this will make the whole thing zero!

So the zeros are $x = 0$ and $x = -2$.

If you don't like this, rewrite the factored form equation as **$(3x + 0)(x + 2) = 0$**

Now set each set of brackets to 0 :)

Special Cases

Solving $ax^2 + bx + c$ by factoring when b = 0.

"Solve" $2x^2 - 18 = 0$

1. Common Factor

$2(x^2 - 9) = 0$

this is still a standard form equation...

$x^2 - 9$ is the same as

$x^2 + 0x - 9$

So... we need to find two numbers that multiply to -9 and add to 0!

Any numbers that add to zero are the same with different signs (they have the same absolute value). So really this question becomes what is the square root of 9? And the answer is, of course 3! So our two numbers are -3 and +3 and our factored form equation is

$$2(x - 3)(x + 3) = 0$$

Graph $y = -3x^2 + 18x - 31$

To graph this function, first we need to **factor it**...

Common Factor?

NOPE

Two numbers that multiply to (-31)(-3) and add to (+18)?

NOPE

We can't factor it!!! Now what?!

And don't say Quadratic Formula!

Complete the Square!

Complete the square.

$$y = 20x - 2.5x^2 - 35$$

In order to ***complete the square***, we need a function in the form

$$y = ax^2 + bx + c$$

our function is mixed up. First let's rearrange it.

$$y = -2.5x^2 + 20x - 35$$

That's better. Now, because $a \neq 1$, we need to factor ***a*** out of the first two terms.

$$y = -2.5[x^2 - 8x] - 35$$

Notice that our ***c*** term is just hanging out at the end. He'll be there until pretty much the very end!

Next, we need to look at our binomial in the square brackets and determine what constant needs to be added to create a perfect square trinomial.

Complete the Square

Remember: there is a trick!

just take half the ***b*** value then square it!

Half of -8 is -4... $(-4)^2$ is 16!

Therefore, we need to add **<u>and</u>** subtract 16 to make sure we don't change anything.

$$y = -2.5[x^2 - 4x + 16 - 16] - 35$$

Now we identify the Perfect Square Trinomial.

$$y = -2.5[(x^2 - 4x + 16) - 16] - 35$$

Next we factor it.

$$y = -2.5[(x - 4)^2 - 16] - 35$$

Complete the Square

Finally, we multiply our ***a*** value back through the brackets and simplify!

$$y = -2.5[(x-4)^2 - 16] - 35$$

$$y = -2.5(x-4)^2 + 40 - 35$$

$$\boxed{y = -2.5(x-4)^2 + 5}$$

If we wanted, we could now quite easily graph our function with a vertex of (4, 5) and a step pattern multiplied by -2.5.

Graph $y = -3x^2 + 18x - 31$

To graph this function, first we need to **complete the square**.

$y = -3(x^2 - 6x) - 31$

$y = -3(x^2 - 6x + 9 - 9) - 31$

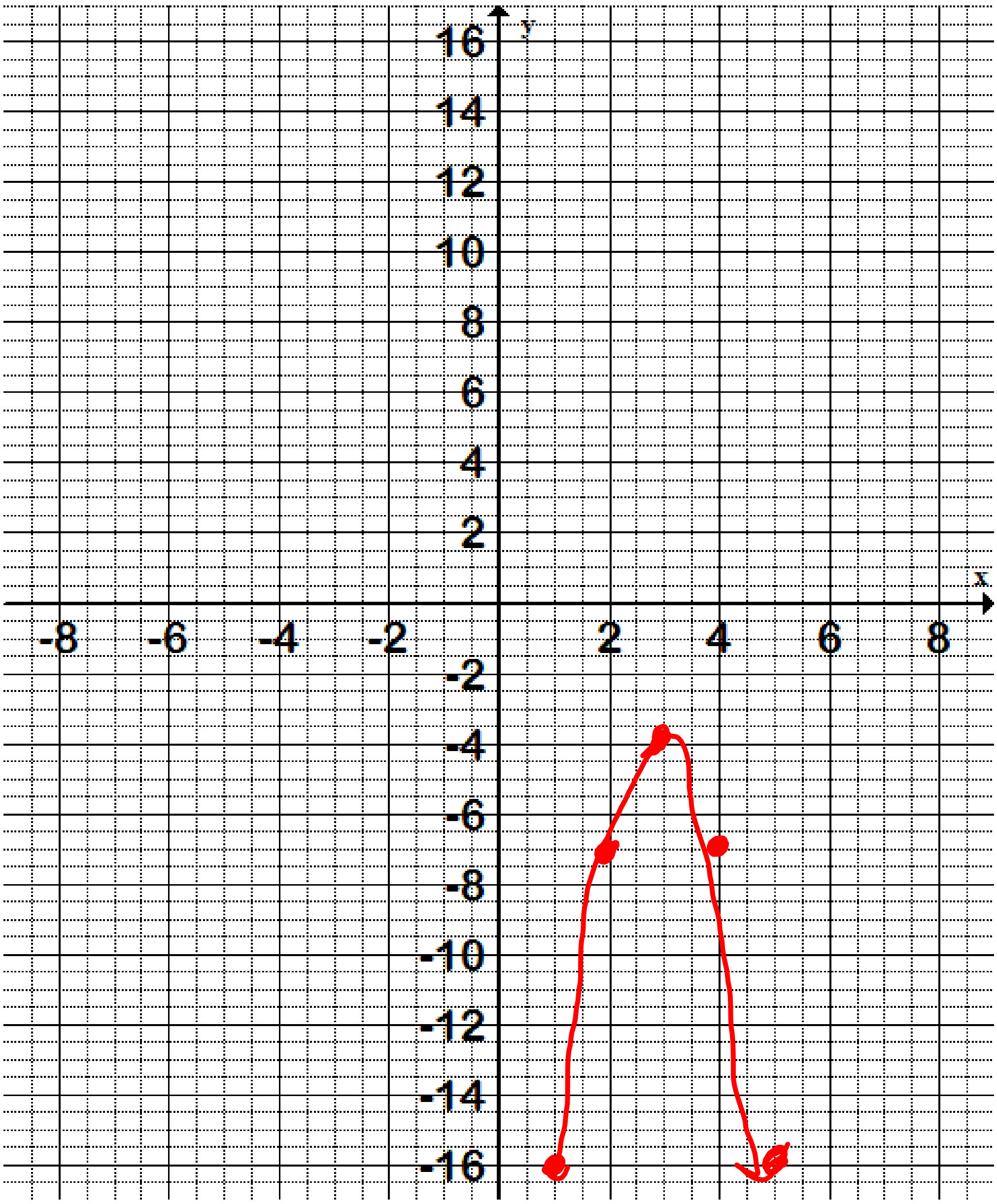
$y = -3((x-3)^2 - 9) - 31$

$y = -3(x-3)^2 + 27 - 31$

$y = -3(x-3)^2 - 4$

Action!

$$y = -3x^2 + 18x - 31$$



Consolidation

Interesting Problems

Determine the **equation of the quadratic relation** in vertex form that has a vertex of (-1,7) and passes through the point (-4, 25).

Consolidation

Interesting Problems

Determine the **equation of the quadratic relation** in vertex form represented by the Table of Values below.

x	y
-11	-17
-9	-11
-7	-9
-5	-11
-3	-17
-1	-27
1	-41
3	-59
5	-81