Solutions

Date: _______________

Introduction to Functions – Review

When we are dealing with transformations, we need our functions in the form;

$$g(x) = af[k(x-d)] + c$$

c (not x)

1. For $f(x) = |x|$,
   a. Graph $f(x)$ on the grid below.
   b. Determine the domain and range of $f(x)$.
   c. List the transformations that must be applied to $f(x)$, to obtain $g(x) = -2f(-2x-8)+7$.
   d. Graph $g(x)$ on the grid below.
   e. Determine the domain and range of $g(x)$.
   f. Determine the value of $f(-3) + g(-5)$. → look at the graph! $f(-3)=3$, $g(-5)=3$

$\boxed{g(x) = -2f[-2(x+4)]+7}$

b) $f(x)$

domain $= \{x \in \mathbb{R}\}$

range $= \{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$

c) $a=-2 \Rightarrow$ vertical stretch by a factor of 2, reflect in x-axis

$k=-2 \Rightarrow$ horizontal compression by a factor of $\frac{1}{2}$, reflect in y-axis

$d=-4 \Rightarrow$ horizontal shift left 4 units

$c=7 \Rightarrow$ vertical shift up 7 units

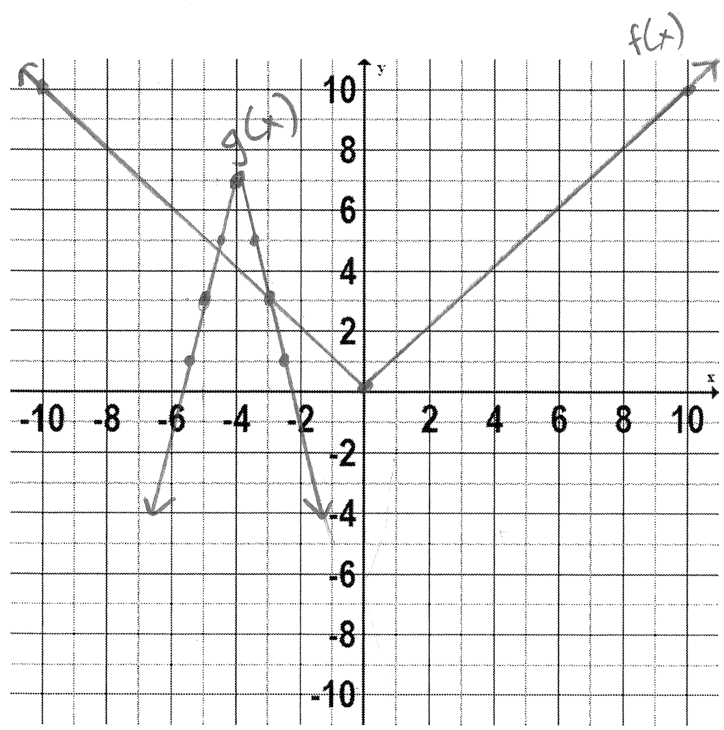
d)

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

x-values: $(\div -2) - 4$

y-values: $(\times -2) + 7$

x	y
-2.5	1
-3	3
-3.5	5
-4	7
-4.5	5
-5	3
-5.5	1



e) $g(x)$

domain $= \{x \in \mathbb{R}\}$

range $= \{g(x) \in \mathbb{R} \mid g(x) \leq 7\}$

f) $f(-3) + g(-5)$

$= [|-3|] + [-2|-2((-5)+4)|+7]$

$= [3] + [-2|-2(-1)|+7]$

$= [3] + [-2|2|+7]$

$= [3] + [-4+7] \longrightarrow = 3+3 = 6$

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2. For $f(x) = x^2$,
   a. Graph $f(x)$ on the grid below.
   b. Determine the domain and range of $f(x)$.
   c. List the transformations that must be applied to $f(x)$, to obtain $g(x) = -f(4x + 8)$.
   d. Graph $g(x)$ on the grid below.
   e. Determine the domain and range of $g(x)$.
   f. Determine the equation of $g^{-1}(x)$.
   g. Graph $g^{-1}(x)$ on the grid below.

$g(x) = -f(4(x+2))$

$a=-1 \quad k=4 \quad d=-2$

b) f(x)

domain $= \{x \in \mathbb{R}\}$

range $= \{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$

c) $a=-1 \Rightarrow$ ~~ver~~ flipped in vertical axis

$k=4 \Rightarrow$ horizontal compression by factor of $\frac{1}{4}$

$d=-2 \Rightarrow$ horizontal shift left 2 units

d)

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

x-values $(\div 4) - 2$ $\longrightarrow$

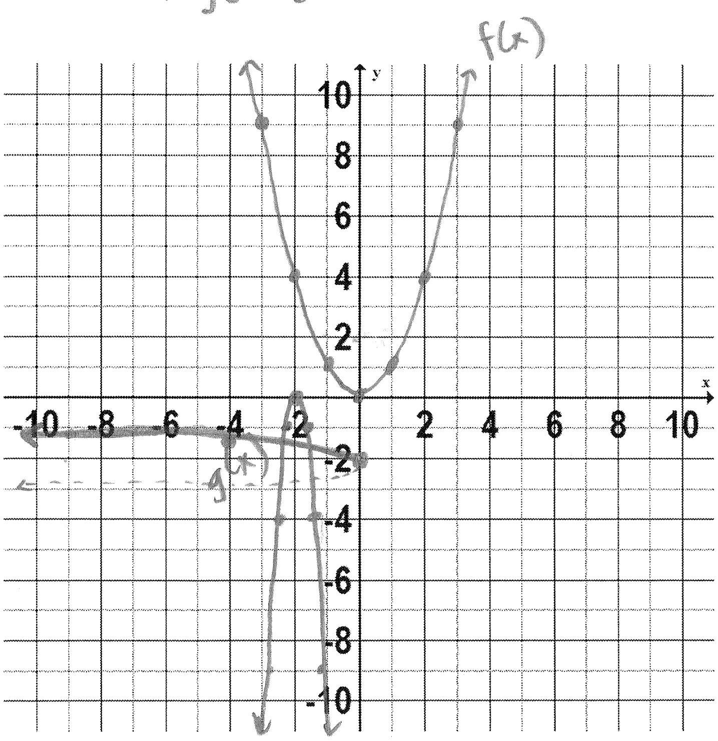
y-values $(\times -1)$

x	y
-2.75	-9
-2.5	-4
-2.25	-1
-2	0
-1.75	-1
-1.5	-4
-1.25	-9

e) g(x)

domain $= \{x \in \mathbb{R}\}$

range $= \{g(x) \in \mathbb{R} \mid g(x) \leq 0\}$



f) $y = -(4(x+2))^2$

$$\frac{x}{-1} = \frac{-(4(y+2))^2}{-1}$$

$$\sqrt{-x} = \sqrt{(4(y+2))^2}$$

$$\frac{4(y+2)}{4} = \frac{\sqrt{-x}}{4}$$

$$\underset{-2}{y+2} = \frac{\sqrt{-x}}{4} - 2$$

$$y = \frac{\sqrt{-x}}{4} - 2$$

OR

$$\boxed{g^{-1}(x) = \frac{1}{4}\sqrt{-1(x)} - 2}$$

not flipped upside down, graph top half

* $\boxed{x \leq 0}$

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3. For $f(x) = \sqrt{x}$,
   a. Graph $f(x)$ on the grid below.
   b. Determine the domain and range of $f(x)$.
   c. List the transformations that must be applied to $f(x)$, to obtain $g(x) = -f(-3x+3)+4$.
   d. Graph $g(x)$ on the grid below.
   e. Determine the domain and range of $g(x)$.
   f. Determine the value of $g(-2) - f(4)$.
   g. Determine the equation of $g^{-1}(x)$.
   h. Graph $g^{-1}(x)$ on the grid below.

$g(x) = -f[-3(x-1)]+4$

$a=-1 \quad k=-3 \quad d=1 \quad c=4$

b) $f(x)$
domain $= \{x \in \mathbb{R} \mid x \geq 0\}$ range $= \{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$

c) $a=-1 \Rightarrow$ flipped on x-axis
$k=-3 \Rightarrow$ horizontal compression by a factor of $\frac{1}{3}$, reflected in y-axis
$d=1 \Rightarrow$ horizontal shift right 1 unit
$c=4 \Rightarrow$ vertical shift up 4 units

d)

x	y
0	0
1	1
4	2
9	3

x-values: $(\div -3)+1$
y-values: $(x-1)+4$

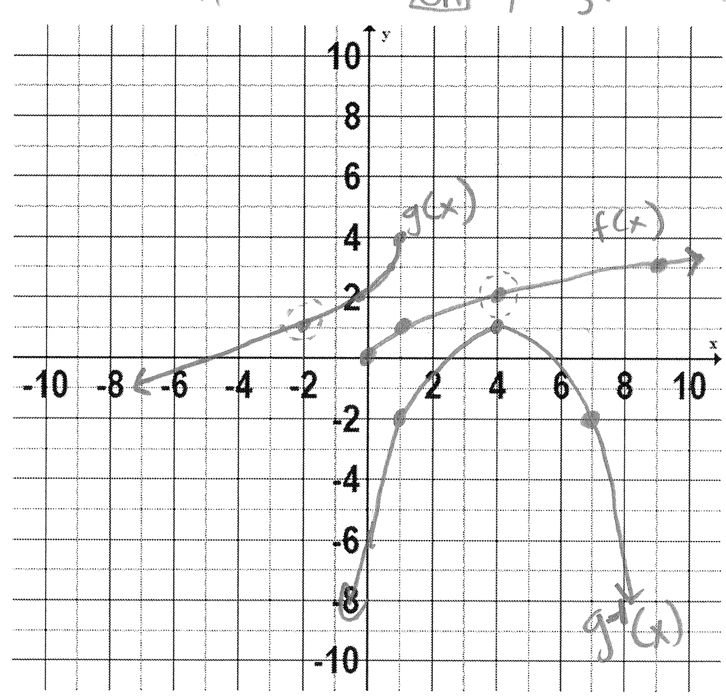
x	y
1	4
2/3	3
-1/3	2
-2	1

e) $g(x)$
domain $= \{x \in \mathbb{R} \mid x \leq 1\}$
range $= \{g(x) \in \mathbb{R} \mid g(x) \leq 4\}$

f) $g(-2) - f(4)$
$= \left(-\sqrt{-3((-2)-1)}+4\right) - \left(\sqrt{4}\right)$
$= \left(-\sqrt{-3(-3)}+4\right) - (2)$
$= \left(-\sqrt{9}+4\right) - (2)$
$= (-3+4) - (2)$
$= (1) - (2)$
$\boxed{= -1}$

* look at graph...
$g(-2) = 1$
$f(4) = 2$

g) $y = -\sqrt{-3(x-1)} + 4$
$x = -\sqrt{-3(y-1)} + 4$
$-4 \qquad -4$
$\frac{x-4}{-1} = \frac{-\sqrt{-3(y-1)}}{-1}$
$(-x+4)^2 = \left(\sqrt{-3(y-1)}\right)^2$
$\frac{(-x+4)^2}{-3} = \frac{\cancel{-3}(y-1)}{\cancel{-3}}$
$\frac{(-x+4)^2}{-3} = y-1$
$+1 \qquad +1$
$\Rightarrow y = \frac{(-x+4)^2}{-3} + 1$
$\boxed{OR}$ $y = -\frac{1}{3}(-1(x-4))^2 + 1$



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4. For $f(x) = x^2$,
   a. Graph $f(x)$ on the grid below.
   b. Determine the domain and range of $f(x)$.
   c. List the transformations that must be applied to $f(x)$, to obtain $g(x) = f(3 - x) + 3$.
   d. Graph g(x) on the grid below.
   e. Determine the domain and range of $g(x)$.
   f. Determine and list any invariant points on your graph.
   g. Determine the equation of $g^{-1}(x)$.
   h. Graph $g^{-1}(x)$ on the grid below.
   i. Determine the value of $f(2) - g(2)$.

$g(x) = f(-1(x-3)) + 3$

$k=-1 \quad d=3 \quad c=3$

b) domain $= \{x \in \mathbb{R}\}$ range $= \{f(x) \in \mathbb{R} \mid f(x) \geq 0\}$

c) $k=-1 \Rightarrow$ ~~ref~~ reflected in y-axis
$d=3 \Rightarrow$ horizontal shift right 3 units
$c=3 \Rightarrow$ vertical shift up 3 units

f) There is an invariant point at (2, 4)

d)

X	Y
-3	9
-2	4
-1	1
$\boxed{0}$	$\boxed{0}$
1	1
2	4
3	9

x-values $(\div -1) + 3$ $\Longrightarrow$ y-values $+3$

X	Y
6	12
5	7
4	4
$\boxed{3}$	$\boxed{3}$
2	4
1	7
0	12

e) domain $= \{x \in \mathbb{R}\}$
range $= \{g(x) \in \mathbb{R} \mid g(x) \geq 3\}$

g) $y = (-1(x-3))^2 + 3$

$x = (-1(y-3))^2 + 3$
$-3 \qquad\qquad -3$

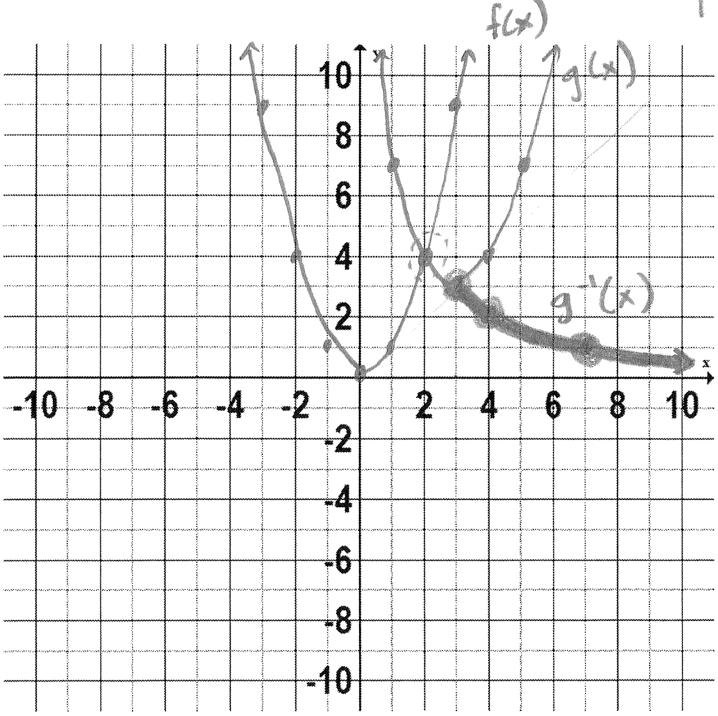
$x - 3 = (-1(y-3))^2$

$\dfrac{\sqrt{x-3}}{-1} = \dfrac{-1(y-3)}{-1}$

$-\sqrt{x-3} = y - 3$
$+3 \qquad +3$

$\boxed{y = -\sqrt{x-3} + 3}$

(graph the bottom half!)
(flipped upside down)



h) $f(2) - g(2) = 0$
(look at the graphs!)

$= (2)^2 - ((-1((2)-3))^2 + 3)$
$= (4) - ((-1(-1))^2 + 3)$
$= (4) - (1+3)$
$= 4-4$
$= 0$