Learning Goal: I will solve exponential equations.

Minds On: 1. Graphs of exponential functions in real life
2. Whiteboards - solve it!

Action: Solving Exponential Equations

Consolidation: Exit Question - whiteboards or exit cards...depends on time

Minds On

What's the Equation?

You start with M_0 dollars in the bank, the amount of money you have in the bank:

a. Doubles every year.

$M(t) = M_0 \times 2^t$

b. Increases by 75% every year.

$M(t) = M_0 \times 1.75^t$

c. Increases by 80% every 3 years.

$M(t) = M_0 \times 1.80^{t/3}$

d. Decreases by 15% every 7 years.

$M(t) = M_0 \times 0.85^{t/7}$

Minds On

Half-Life

All radioactive substances decrease in mass over time.

What is the general exponential equation for half life?

$$M(t) = M_0 \times 0.5^{t/h}$$

Minds On

Half-Life

$M(t) = M_0 \times 0.5^{\frac{t}{30}}$

Example

The half life of cesium-137, a radioactive substance released by the Chernobyl accident (1986), is 30 years.

If 4 kg of the substance was released in the accident,

a. How much remains today? → $t = 31$

$M(t) = 4 \times 0.5^{\frac{31}{30}}$

$M(t) = 1.95 kg$

b. How much will remain in 2030? → $t = 44$

$M(t) = 4 \times 0.5^{\frac{44}{30}}$

$M(t) = 1.45 kg$

c. When will there be less than 1 kg remaining?

~~$1 = 4 \times 0.5^{\frac{t}{30}}$~~ half of 4 = 2 } cut in half time
half of 2 = 1 } 60 yrs

d. When will there be less than 200 g remaining?

Action

Solving Exponential Equations

Example 1: Different strategies to solve an exponential equation

All radioactive substances decrease in mass over time. Kristen works in a laboratory that uses radioactive substances. T he laboratory received a shipment of 200 g of radioactive radon, and 16 days later, 12.5 g of the radon remained. What is the half-life of radon?

Solution A

$$M(t) = M_0 \times 0.5^{\frac{t}{h}}$$

$$12.5 = 200 \times 0.5^{\frac{16}{h}}$$

$$\frac{12.5}{200} = 0.5^{\frac{16}{h}}$$

$$\frac{1}{16} = \frac{1}{2}^{\frac{16}{h}}$$

$$\frac{1}{2}^{4} = \frac{1}{2}^{\frac{16}{h}}$$

$$4 = \frac{16}{h}$$

$$h$$

$$\boxed{h = 4}$$

Action

Solution B

$$M(t) = M_0 \times 0.5^{\frac{t}{h}}$$

$$\frac{12.5}{200} = \frac{200 \times 0.5^{\frac{16}{h}}}{200}$$

$$0.0625 = 0.5^{\frac{16}{h}}$$

$$\log 0.0625 = \log 0.5^{\frac{16}{h}}$$

$$\log 0.0625 = \frac{16}{h} \log 0.5$$

$$\frac{16}{h} = \frac{\log 0.0625}{\log 0.5}$$

$$\frac{16}{h} = 4$$

$$h = 4$$

Action

Solution C

- graph both sides
- find POI

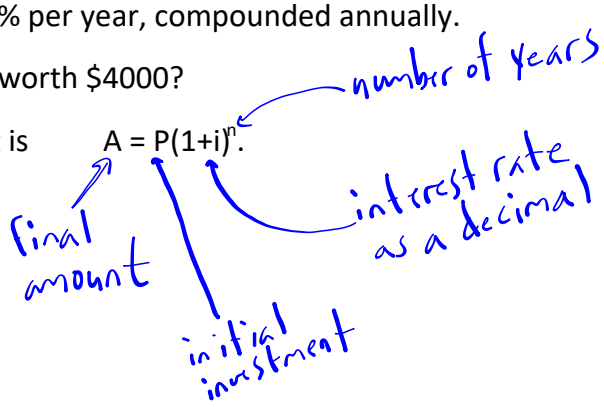
Action

Example 2: Using Logs to solve a problem

An investment of $2500 grows at a rate of 4.8% per year, compounded annually.

How long will it take for the investment to be worth $4000?

Recall that the formula for compound interest is $A = P(1+i)^n$.



$A = P(1+i)^n$

$\frac{4000}{2500} = \frac{2500(1+0.048)^n}{2500}$

$\log 1.6 = \log 1.048^n$

$\log 1.6 = n \log 1.048$

$n = \frac{\log 1.6}{\log 1.048}$

$n = 10.02$ years

Action

Example 3: Exponentials with more than one power

Solve $2^{x+2} - 2^x = 24$

$2^x(2^2 - 1) = 24$

$2^x(4-1) = 24$

$\frac{2^x(3)}{3} = \frac{24}{3}$

$2^x = 8$

$x = 3$

OR

$\log 2^x = \log 8$

$x \log 2 = \log 8$

$x = \frac{\log 8}{\log 2}$

$x = 3$

Action

Example 4: When the exponents have different bases

Solve $2^{x+1} = 3^{x-1}$ to three decimal places.

$$\log 2^{x+1} = \log 3^{x-1}$$

$$(x+1)\log 2 = (x-1)\log 3$$

$$x\log 2 + \log 2 = x\log 3 - \log 3$$

$$-x\log 3 \quad -\log 2 \qquad -x\log 3 \quad -\log 2$$

$$x\log 2 - x\log 3 = -\log 2 - \log 3$$

$$\frac{x(\log 2 - \log 3)}{\log 2 - \log 3} = \frac{-\log 2 - \log 3}{\log 2 - \log 3}$$

$$x = \frac{-\log 2 - \log 3}{\log 2 - \log 3}$$

$$x = 4.419$$

Consolidation

Solve: $9^{2x + 1} = 81(27^{x})$

Consolidation

Practice

Pg. 485

1 - 3 (a few from each)

4, 5, 8, 10, 11