Learning Goal: I will be able to make connections between the laws of exponents and the laws of logarithms.

Minds On: Exponent Laws

Action: Log Laws

Consolidation: page 466 Practice + Exit Card

Minds On

Exponent Laws

In Grade 9, you learned the Exponent Laws

The Product Rule

$$(a^x)(a^y) = a^{x+y}$$

The Quotient Rule

$$\frac{a^x}{a^y} = a^{x-y}$$

The Power of a Power Rule

$$(a^x)^y = a^{xy}$$

Action

Mini-Investigation

Complete the table below using your calculator:

x	**log x** (to two decimal places)
2	
4	
8	
16	
32	
64	
128	
256	

Action

Mini-Investigation

Complete the table below using your calculator:

x	**log x** (to two decimal places)
2	0.30
4	0.60
8	0.90
16	1.20
32	1.51
64	1.81
128	2.11
256	2.41

1. Given the information in the table above, attempt to determine a formula for $\log_a(mn)$.

$\log 8 = 0.90 \rightarrow \log(4 \times 2) = 0.90$

$\log 4 = 0.60$

$\log 2 = 0.30$

$\log 8 = \log 2 + \log 4$

$$\log_a(mn) = \log_a m + \log_a n$$

2. Given the information in the table above, attempt to determine a formula for $\log_a\left(\frac{m}{n}\right)$.

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$\log 2 = 0.30$

$\log 4 = 0.60$

$\log 8 = 0.90$

$\log_a\left(\frac{8}{4}\right) = \log_a(2)$

$= \log_a 8 - \log_a 4$

3. Given the information in the table above, attempt to determine a formula for $\log_a(m)^n$.

$$\log_a(m)^n = n \log_a m$$

Product Law of Logarithms

$$\log_a(mn) = \log_a m + \log_a n$$

Quotient Law of Logarithms

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Power Law of Logarithms

$$\log_a(m)^n = n \log_a m$$

Consolidation

Exponents vs. Logarithms

Law	Exponents	Logarithms
Products	$a^m x\, a^n = a^{m+n}$	$\log_a(mn) = \log_a m + \log_a n$
Quotients	$\left(\frac{a^m}{a^n}\right) = a^{m-n}$	$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
Powers	$\left(a^m\right)^n = a^{mn}$	$\log_a m^n = n \cdot \log_a m$
Zero Exponents	$a^0 = 1$	$\log_a 1 = 0$
Negative Exponents	$a^{-m} = \left(\frac{1}{a^m}\right)$	$\log_a\left(\frac{1}{a^m}\right) = -m$

Consolidation

Applying the Laws

Fully simplify each logarithmic expression.

$\log_3 6 + \log_3 4.5$

$= \log_3 (6 \times 4.5)$

$= \log_3 (27)$

$= 3$

$\log_2 48 - \log_2 3$

$= \log_2 \left(\frac{48}{3}\right)$

$= \log_2 (16)$

$= 4$

$$\log_5 \sqrt[3]{25}$$

$$= \log_5 (25)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log_5 (25)$$

$$= \frac{1}{3} (2)$$

$$= \frac{2}{3}$$

Use the properties of logarithms to express $\log_a \sqrt{\frac{x^3y^2}{w}}$ in terms of $\log_a x$, $\log_a y$ and $\log_a w$.

$$\log_a \sqrt{\frac{x^3y^2}{w}}$$

$$= \log_a \left(\frac{x^3y^2}{w}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\log_a\left(\frac{x^3y^2}{w}\right)$$

$$= \frac{1}{2}\left(\log_a x^3 + \log_a y^2 - \log_a w\right)$$

$$= \frac{1}{2}\left(3\log_a x + 2\log_a y - \log_a w\right)$$

$$= \frac{3}{2}\log_a x + \log_a y - \frac{1}{2}\log_a w$$

Consolidation

Practice

Pg. 475

2ef, 3, 4

6 - 10 (a few from each)