Learning Goal: I will solve linear and quadratic trig equations.

Minds On: Solving equations without trig, how do periods affect our answers?

Action: Solving Linear Equations - Note

Consolidation: Exit Question

Minds On

Part 1: Solve Each Equation, Find All Values

3(x + 1) + 5 = 2

$$3(x+1) = -3$$
$$x + 1 = -1$$
$$x = -2$$

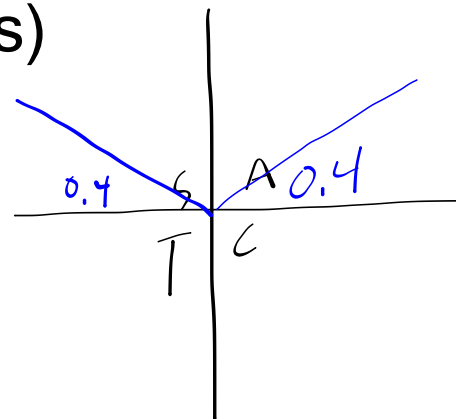$3x^2 + 1 = 13$

$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

sin x = 0.4 (find in radians)

$$x = \sin^{-1}(0.4)$$
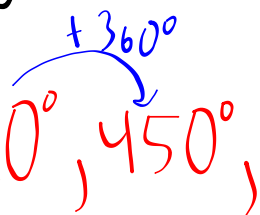
$$x = 0.41$$

$$\text{and } x = \pi - 0.41$$
$$x = 2.73$$

Minds On

Part 2: Looking at trig periods
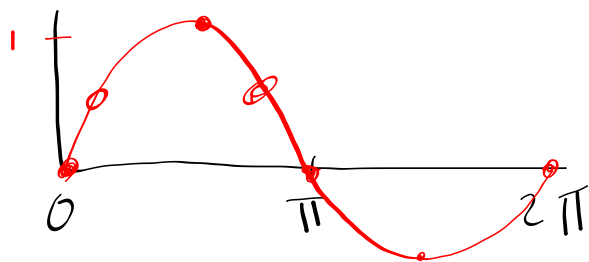
List 5 values where sin x = 1
How did you find them?

$$x = 90°, 450°,$$

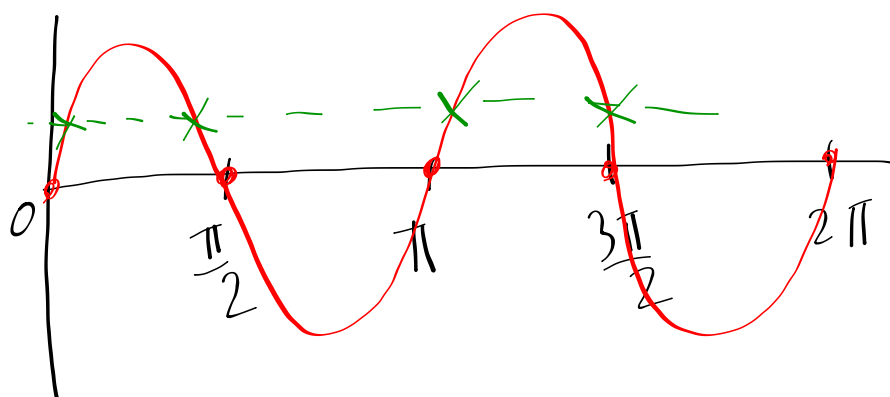$$x = \frac{\pi}{2}, \frac{5\pi}{2},$$

$+ 360°$

$+ 2\pi$

add the period each time

Minds On

Part 3: How many times?

How many times will sin(x) = 0.5 between 0 and 2π?

2

How many times will sin(2x) = 0.5 between 0 and 2π?
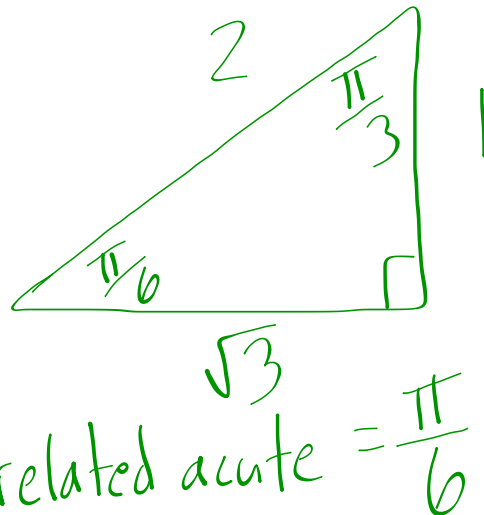
4

Action

Solving Linear Trig Equations

Example 1: You are given the equation $2\sin x + 1 = 0$, $0 \leq x \leq 2\pi$
a) Determine all the solutions in the specified interval.
b) Verify the solutions using graphing technology

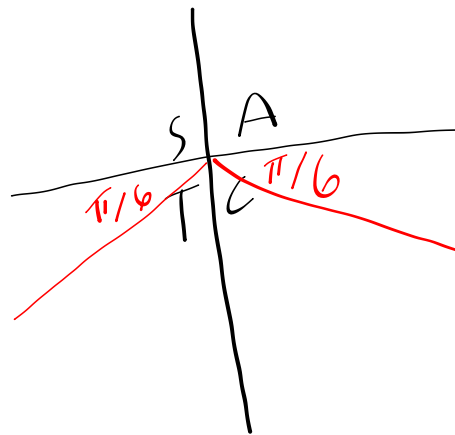$2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = \dfrac{-1}{2}$

related acute $= \dfrac{\pi}{6}$

$x = \pi + \dfrac{\pi}{6}$

$= \dfrac{7\pi}{6}$

$x = 2\pi - \dfrac{\pi}{6}$

$= \dfrac{11\pi}{6}$

Action

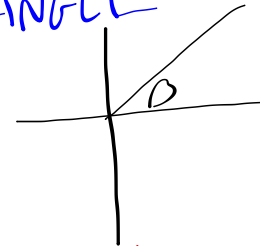**Example 2:** Solve 3(tanx + 1) = 2 where $0 \leq x \leq 360°$,
to 1 decimal place

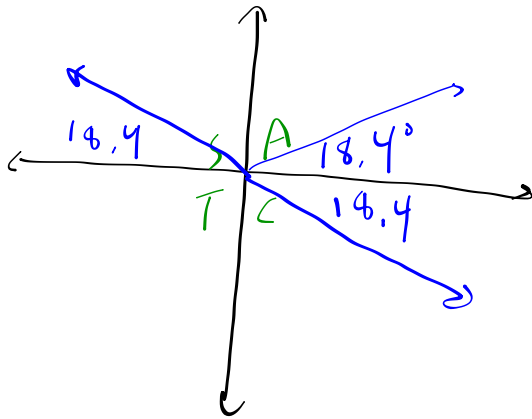$$\frac{3(\tan x + 1)}{3} = \frac{2}{3}$$

$$\tan x + 1 = \frac{2}{3}$$

$$\tan x = -\frac{1}{3}$$

NOT SPECIAL TRIANGLE

Find related acute angle: $\tan x = \frac{1}{3}$

$$x = \tan^{-1}\left(\frac{1}{3}\right)$$

$$x = 18.4°$$

18.4   A 18.4°
T C 18.4

$$x = 180 - 18.4$$
$$= 161.6°$$

$$x = 360 - 18.4$$
$$= 341.6°$$

Action

Example 3: Today, the high tide in Matthews Cove, New Brunswick, is at midnight. The water level at high tide is 7.5 m. The depth, d metres, of the water in the cove at time t hours is modelled by the equation $d(t) = 4 + 3.5\cos(\frac{\pi}{6}t)$. Jenny is planning a day trip to the cove tomorrow, but the water need to be at least 2 m deep for her to manoeuvre her sailboat safely. How can Jenny determine the times when it will be safe for her to sail into Matthews cove?
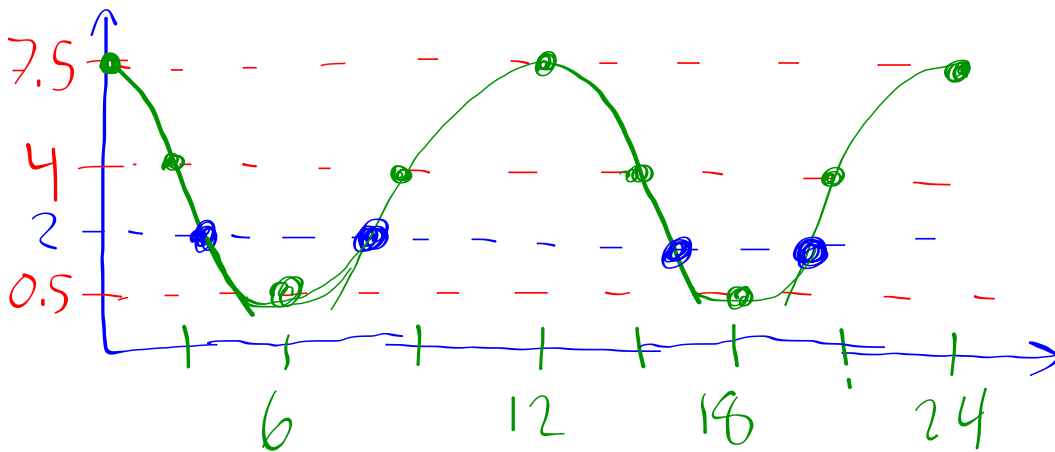
$$period = \frac{2\pi}{\frac{\pi}{6}} = 12$$

$$d(t) = 4 + 3.5\cos\left(\frac{\pi}{6}t\right)$$

$$2 = 4 + 3.5\cos\left(\frac{\pi}{6}t\right)$$
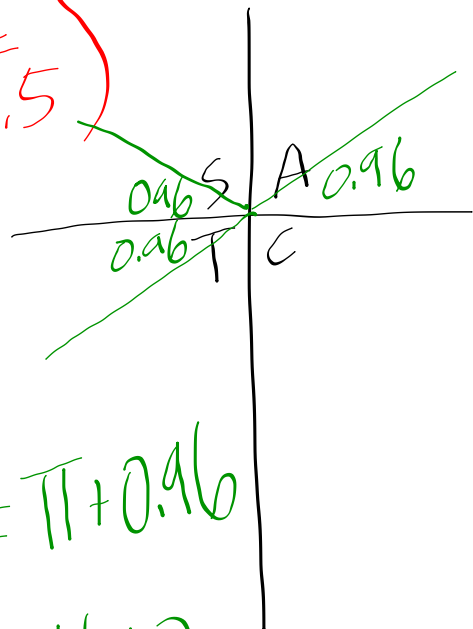
$$2 = 4 + 3.5 \cos\left(\frac{\pi}{6} t\right)$$

$$-2 = 3.5 \cos\left(\frac{\pi}{6} t\right)$$

$$\cos\left(\frac{\pi}{6} t\right) = -\frac{2}{3.5}$$

Find related acute angle

$$\frac{\pi}{6} t = \cos^{-1}\left(\frac{2}{3.5}\right)$$

$$\frac{\pi}{6} t = 0.96$$

$$\begin{array}{c|c}
0.96 \; S & A \; 0.96 \\
\hline
0.96 \; T & C
\end{array}$$

$$\frac{\pi}{6} t = \pi - 0.96 \qquad \frac{\pi}{6} t = \pi + 0.96$$

$$\frac{\pi}{6} t = 2.14 \qquad \frac{\pi}{6} t = 4.10$$

$$\frac{\pi}{6}t = 2.18$$

$$t = 4.16$$

$$\times 60$$

$$t = 4:10am$$

$$\frac{\pi}{6}t = 4.10$$

$$t = 7.83$$

$$\times 60$$

$$t = 7:50am$$

add the 12hr period to each

t also $= 4:10pm$ and $7:50pm$

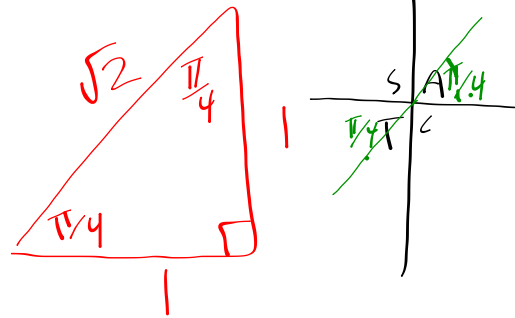She can leave before 4:10am or after 7:50am and return before 4:10pm or after 7:50pm.

Action

Example 4: Solve $2\sin x \cos x = \cos 2x$ for x on $0 \le x \le 2\pi$.

$$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x}$$

$$\tan 2x = 1$$

$$2x = \tan^{-1}(1)$$

$$2x = \frac{\pi}{4} \text{ and } 2x = \frac{5\pi}{4}$$

$$\boxed{x = \frac{\pi}{8} \text{ and } x = \frac{5\pi}{8}}$$

tan x has a period of π

tan 2x has a period of $\frac{\pi}{2}$

To get our other answers, add the period

$$\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4} + \frac{4\pi}{4} = \frac{5\pi}{4}$$

already have it!

$$\frac{5\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4} + \frac{4\pi}{4} = \boxed{\frac{9\pi}{4}}$$

$$\frac{9\pi}{4} + \frac{\pi}{2} = \frac{9\pi}{4} + \frac{4\pi}{4} = \boxed{\frac{13\pi}{4}}$$

$\cancel{\frac{17\pi}{4}}$ ← past 2π, not a solution.

$$\therefore x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

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3, 6, 8, 9, 10, 12

Consolidation

Exit Question

Find two values of x where

$$1 - \sin^2 x = \frac{1}{2}$$