Learning Goal: I will develop and use double angle formulas

Minds On: Same or different?

Action: Double Angle Formulas - Investigation and Practice

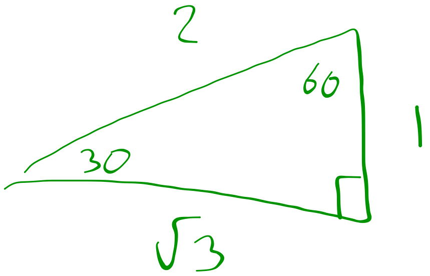
Consolidation: Exit Question

Minds On

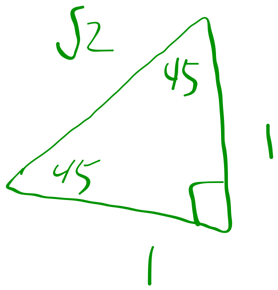
Determine the exact value of the expression below

tan (30° - 45°)

$$\tan(30^\circ - 45^\circ) = \frac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \times \tan 45^\circ}$$



$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\tan 45^\circ = 1$$

$$= \frac{\frac{\sqrt{3}}{3} - 1}{1 + \left(\frac{\sqrt{3}}{3}\right)(1)}$$

$$= \frac{\frac{\sqrt{3}}{3} - \frac{3}{3}}{1 + \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{\sqrt{3} - 3}{3}}{\frac{3 + \sqrt{3}}{3}}$$

$$= \frac{\sqrt{3}-3}{\cancel{3}} \times \frac{\cancel{3}}{3+\sqrt{3}}$$

$$= \frac{\sqrt{3}-3}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{3\sqrt{3}-3-9+3\sqrt{3}}{9\cancel{-3\sqrt{3}}\cancel{+3\sqrt{3}}-3}$$

$$= \frac{-12+6\sqrt{3}}{6}$$

$$\boxed{= -2+\sqrt{3}}$$

How do we write **(sin x)(sin x)**?

$= (\sin x)^2$

$= \sin^2 x$

If $\sin x = \frac{\sqrt{3}}{2}$

$\sin^2 x = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

Minds On

Same or Different?

Describe f(x) = sin(2x) and f(x) = 2 sin(x)

Are they the same or different?

Action

Investigation:

1. Given $\sin 2\theta = \sin(\theta + \theta)$, use the appropriate compound angle formula to expand the right side of the equation. Simplify both sides to develop a formula for $\sin 2\theta$.
2. Verify your double angle formula for sine by graphing each side of the formula as a function.

$$\sin(\theta + \theta) = \sin\theta \times \cos\theta + \cos\theta \times \sin\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

3. Repeat parts 1 and 2 to develop a double angle formula for $\tan 2\theta$.

$$\tan(\theta + \theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \times \tan\theta}$$

$$= \frac{2\tan\theta}{1 - \tan^2\theta}$$

4. Repeat parts 1 and 2 to develop a double angle formula for $\cos 2\theta$.

$$\cos(\theta+\theta) = \cos\theta \times \cos\theta - \sin\theta \times \sin\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

5. Use the Pythagorean Identity to rearrange your formula for $\cos 2\theta$ to create different versions of the double angle formula. Be sure to verify each new formula.

$\sin^2\theta + \cos^2\theta = 1$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\boxed{\sin^2\theta = 1 - \cos^2\theta}$

$\boxed{\cos^2\theta = 1 - \sin^2\theta}$

Rearrangement #1

$$\cos(2\theta) = \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1$$

Rearrangement #2

$$\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$

Action

Example 1: Simplify each of the following expressions and then evaluate.

a) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

What formula is this?

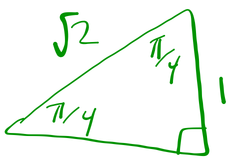
$\sin 2\theta$

$\theta = \frac{\pi}{8}$

$= \sin\left(2\left(\frac{\pi}{8}\right)\right)$

$= \sin\left(\frac{2\pi}{8}\right)$

$= \sin\left(\frac{\pi}{4}\right)$



$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

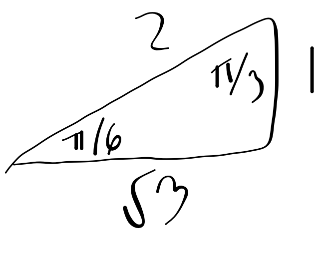
What formula is this?

$\tan 2\theta$

$\theta = \frac{\pi}{6}$

$= \tan\left(2\left(\frac{\pi}{6}\right)\right)$

$= \tan\left(\frac{\pi}{3}\right)$



$= \frac{\sqrt{3}}{1} = \sqrt{3}$

Action

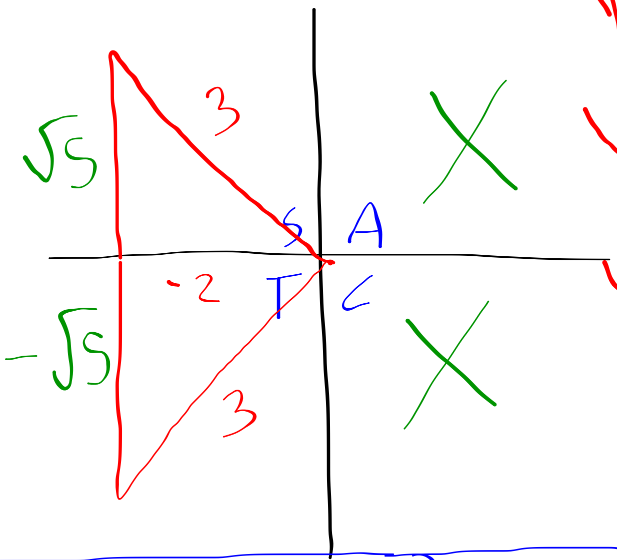
<u>Example 2:</u> If $\cos\theta = -\frac{2}{3}$ and $0 \leq \theta \leq 2\pi$, determine the value of $\cos 2\theta$ and $\sin 2\theta$.

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(-\frac{2}{3}\right)^2 - 1$$
$$= 2\left(\frac{4}{9}\right) - 1$$
$$= \frac{8}{9} - \frac{9}{9}$$
$$= \frac{-1}{9}$$

$\sin 2\theta$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

We need $\sin\theta$



$$y^2 = 3^2 - (-2)^2$$
$$y^2 = 9 - 4$$
$$y^2 = 5$$
$$y = \pm\sqrt{5}$$

Q II

$\cos\theta = -\frac{2}{3}$

$\sin\theta = \frac{\sqrt{5}}{3}$

$$\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$$
$$= 2\left(\frac{-2\sqrt{5}}{9}\right)$$
$$= \frac{-4\sqrt{5}}{9}$$

Q III

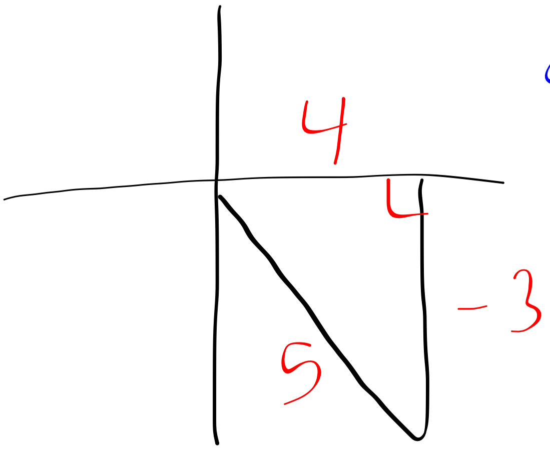
$\cos\theta = -\frac{2}{3}$

$\sin\theta = -\frac{\sqrt{5}}{3}$

$$\sin 2\theta = 2\left(-\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$$
$$= 2\left(\frac{2\sqrt{5}}{9}\right)$$
$$= \frac{4\sqrt{5}}{9}$$

Action

Example 3: Given $\tan\theta = -\frac{3}{4}$ where $\frac{3\pi}{2} \leq \theta \leq 2\pi$, calculate the value of $\cos 2\theta$.



$\cos\theta = \frac{4}{5}$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 2\left(\frac{4}{5}\right)^2 - 1$$
$$= 2\left(\frac{16}{25}\right) - 1$$
$$= \frac{32}{25} - \frac{25}{25}$$
$$\boxed{= \frac{7}{25}}$$

Action

Example 4: Develop a formula for $sin\frac{x}{2}$.

$\theta = \frac{x}{2}$

Use $\cos 2\theta = 1 - 2\sin^2\theta$

$\cos\left(2\left(\frac{x}{2}\right)\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos(x) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$-1 \quad -1$

$\frac{\cos(x) - 1}{-2} = \frac{-2\sin^2\left(\frac{x}{2}\right)}{-2}$

$\sin^2\left(\frac{x}{2}\right) = \frac{\cos(x) - 1}{-2}$

$\sin^2\left(\frac{x}{2}\right) = \frac{-\cos(x) + 1}{2}$

$$\sqrt{\sin^2\left(\frac{x}{2}\right)} = \sqrt{\frac{1-\cos(x)}{2}}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

Consolidation

Determine the value of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$,

given $sin\theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$.

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