Learning Goal: I will identify equivalent trigonometric relationships.

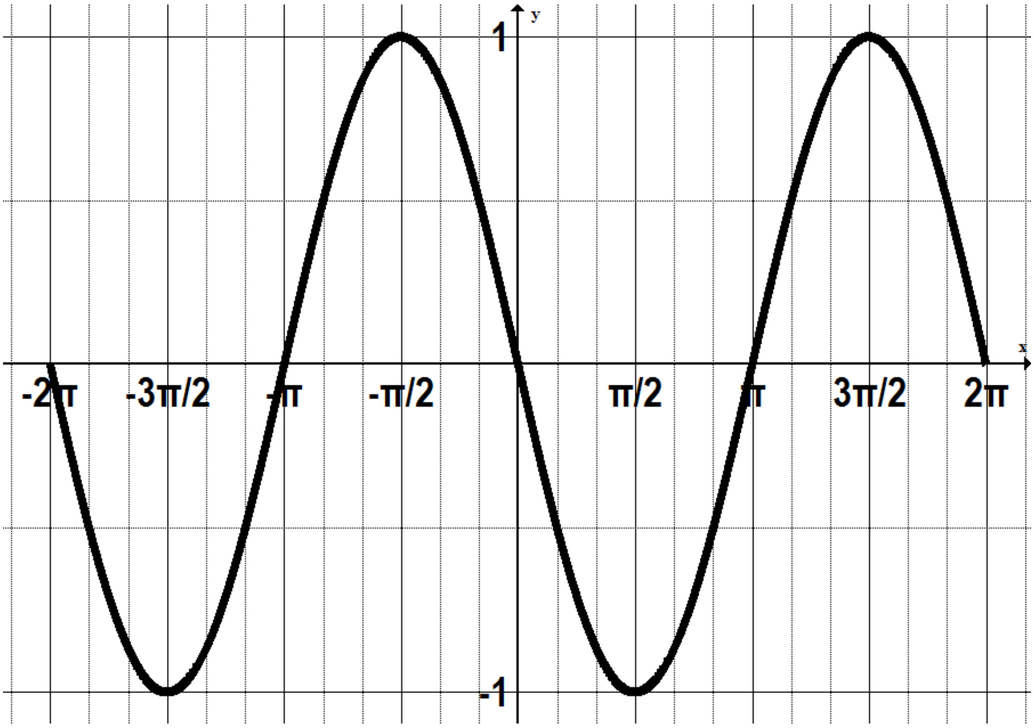
Minds On: What's the Equation?

Action: Equivalent Trigonometric Functions

Consolidation: Are we Equivalent?

Minds On

What's the Equation?



Write 4 possible equations for this graph.

$y = -\sin\theta$

$y = \sin(\theta - \pi)$

$y = \cos\left(\theta + \frac{\pi}{2}\right)$

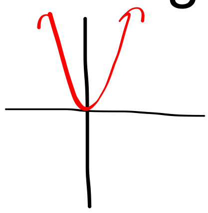
$y = -\cos\left(\theta - \frac{\pi}{2}\right)$

$y = \sin(-\theta)$

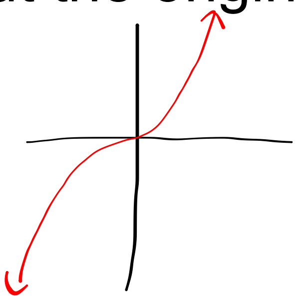
Action

Even and Odd Functions

An **even function** is a function whose graph is symmetrical in the y-axis.



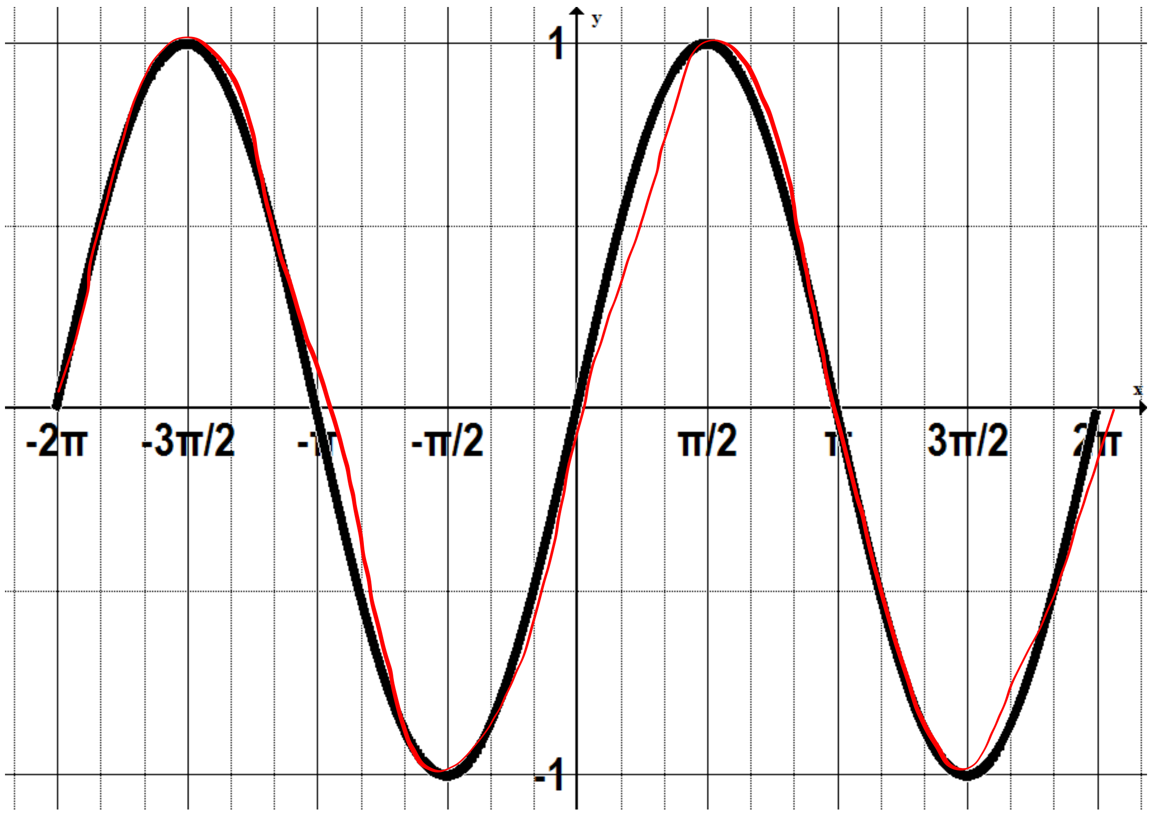
An **odd function** is a function whose graph has rotational symmetry about the origin.



Action

Even and Odd Functions

Is f(θ) = sin θ an even or odd function?



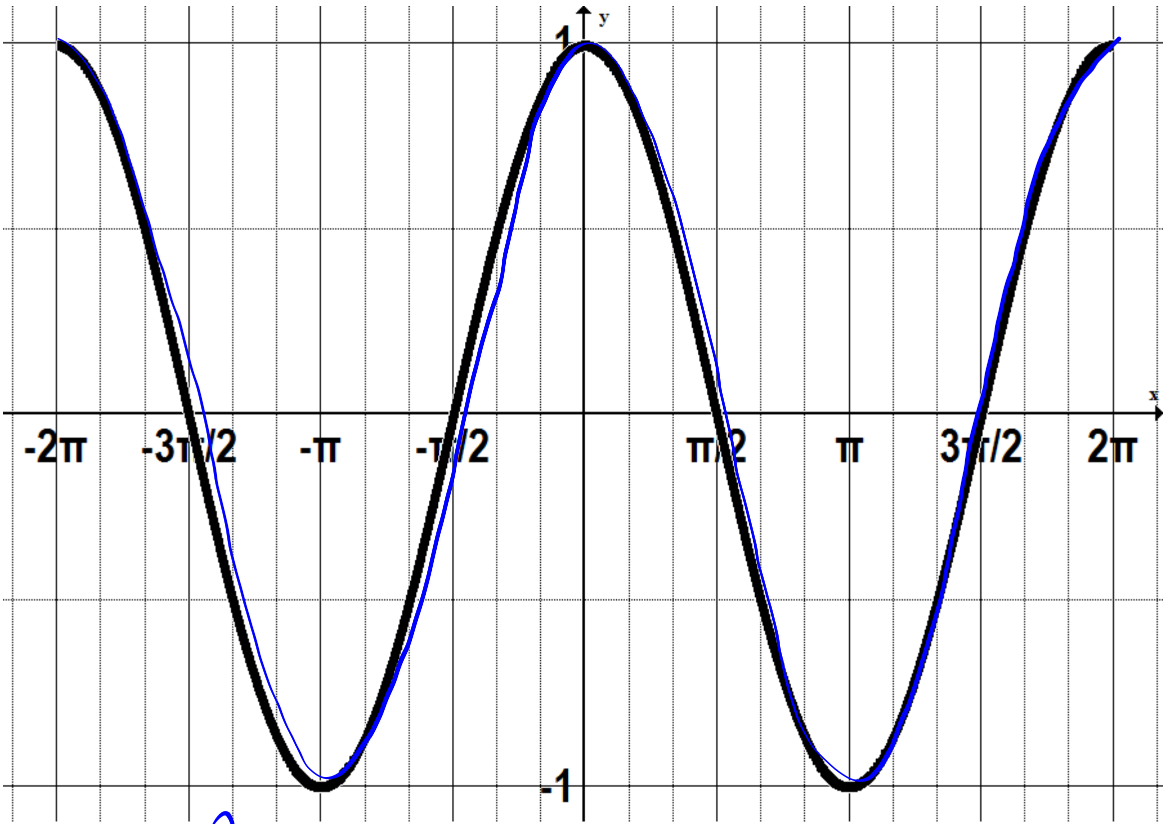
sin θ is odd

This means $\sin\theta = -\sin(-\theta)$

Action

Even and Odd Functions

Is f(θ) = cos θ an even or odd function?



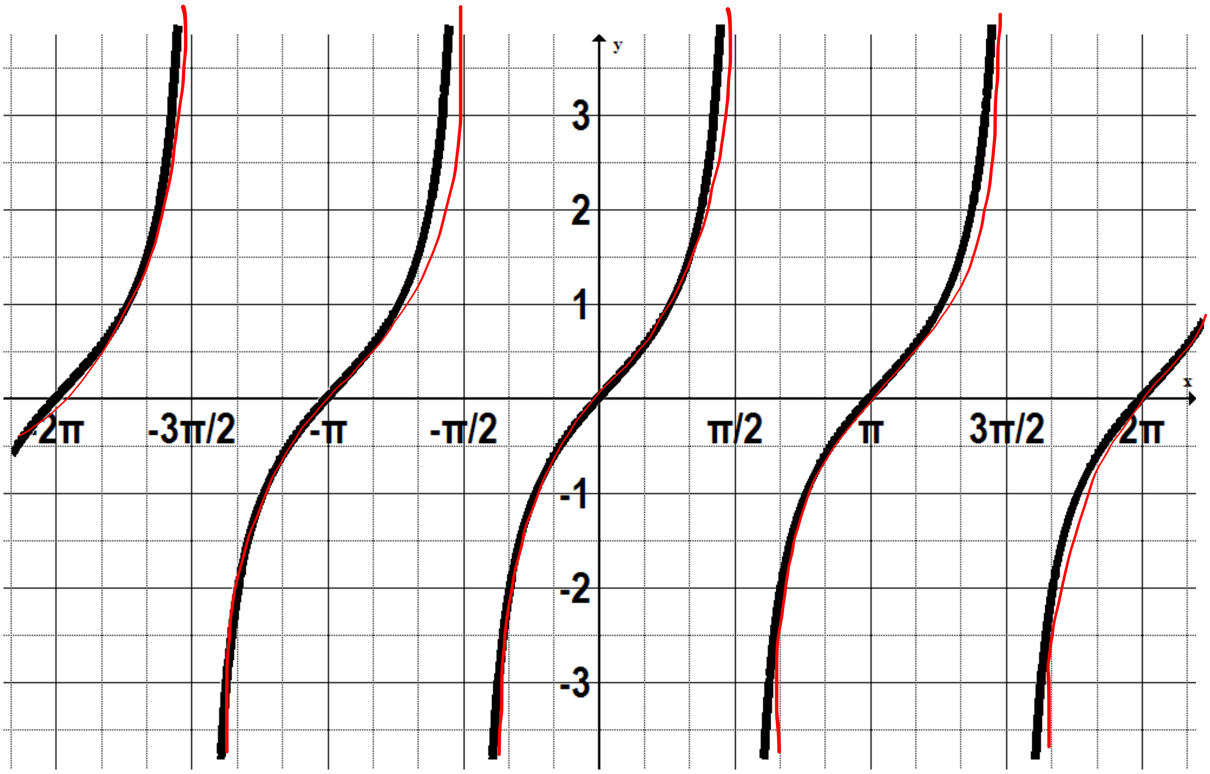
cos θ is even

This means $\cos\theta = \cos(-\theta)$

Action

Even and Odd Functions

Is f(θ) = tan θ an even or odd function?

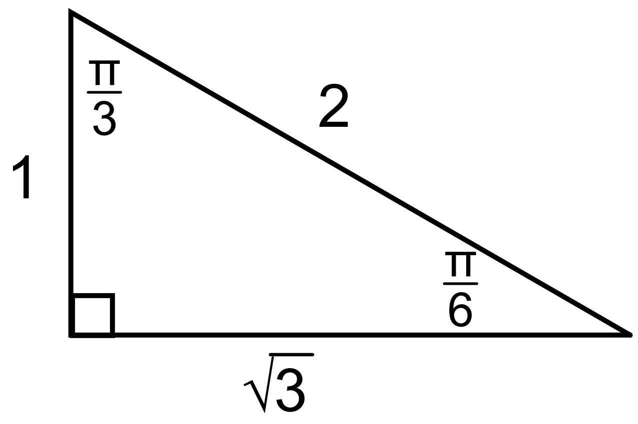


tan θ is odd

This means $\tan\theta = -\tan(-\theta)$

Action

Cofunction Identities

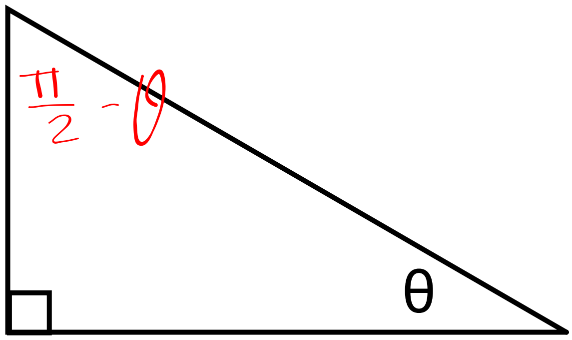


$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\csc\left(\frac{\pi}{6}\right) = \frac{2}{1}$
$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\sec\left(\frac{\pi}{3}\right) = \frac{2}{1}$	$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$
$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$	$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$	$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$	$\cot\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{1}$

What expressions are equivalent?

Action

Cofunction Identities



What is the measure of the missing angle?

Then:

sin θ = $\cos\left(\frac{\pi}{2} - \theta\right)$

cos θ = $\sin\left(\frac{\pi}{2} - \theta\right)$

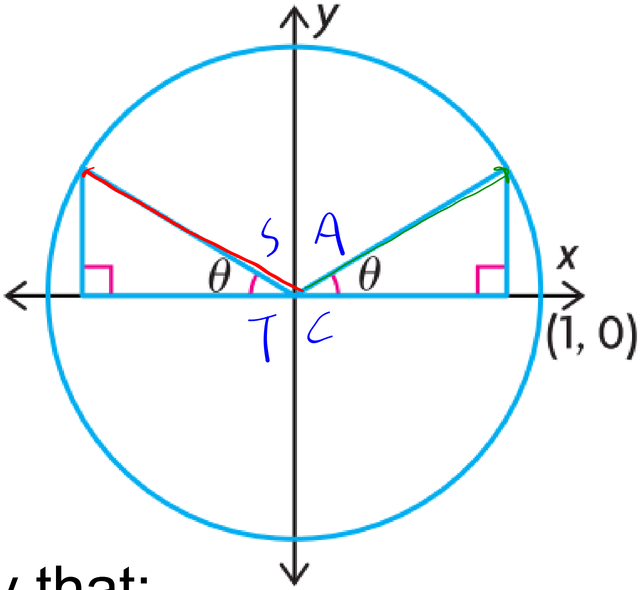
tan θ = $\cot\left(\frac{\pi}{2} - \theta\right)$

Action

Equivalent Expressions from the Plane

With a related acute angle, θ, in quadrant I we can find more equivalent expressions.

Here, the angle of interest is (π - θ).



We can say that:

$\sin(\pi - \theta) =$ sin θ

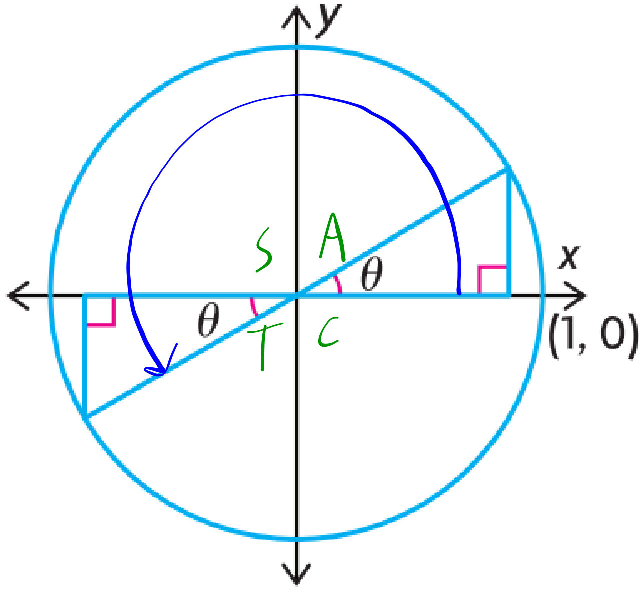
$\cos(\pi - \theta) =$ − cos θ

$\tan(\pi - \theta) =$ − tan θ

Action

Equivalent Expressions from the Plane

Here, the angle of interest is $\pi + \theta$.



We can say that:

$\sin(\pi+\theta) = -\sin\theta$

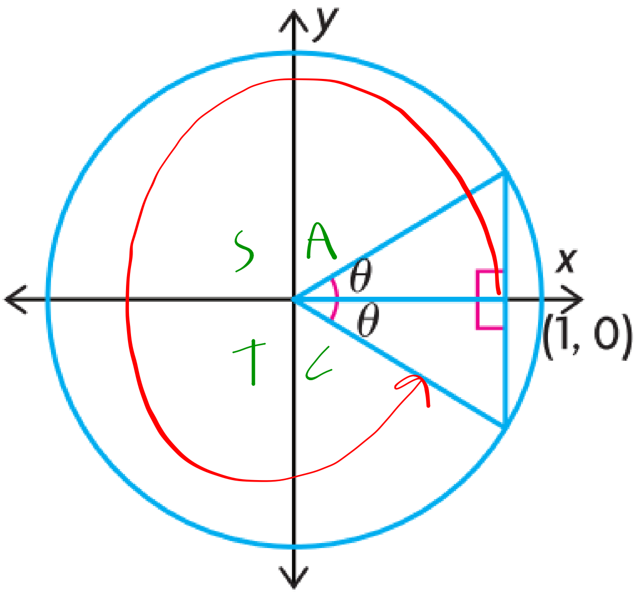
$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = \tan\theta$

Action

Equivalent Expressions from the Plane

Here, the angle of interest is $2\pi - \theta$.



We can say that:

$\sin(2\pi - \theta) = -\sin\theta$

$\cos(2\pi - \theta) = \cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

1. Horizontal Translations

   by multiples of the period $\sin\theta = \sin(\theta \pm 2\pi)$

   by shifting sin and cos by pi/2

2. Reflecting **even functions** across y-axis

   cos x = cos -x

3. Reflecting **odd functions** across x-axis and y-axis

   sin x = - sin -x

   - sin x = sin -x

   tan x = - tan -x

   -tan x = tan -x

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

4. Using the cofunction identities

$\sin \theta = \cos (\pi/2 - \theta)$

$\cos \theta = \sin (\pi/2 - \theta)$

$\tan \theta = \cot (\pi/2 - \theta)$

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

5. Using principle and related acute angles

In Quadrant II

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

In Quadrant III

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

$\tan(\pi + \theta) = \tan\theta$

In Quadrant IV

$\sin(2\pi - \theta) = -\sin\theta$

$\cos(2\pi - \theta) = \cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

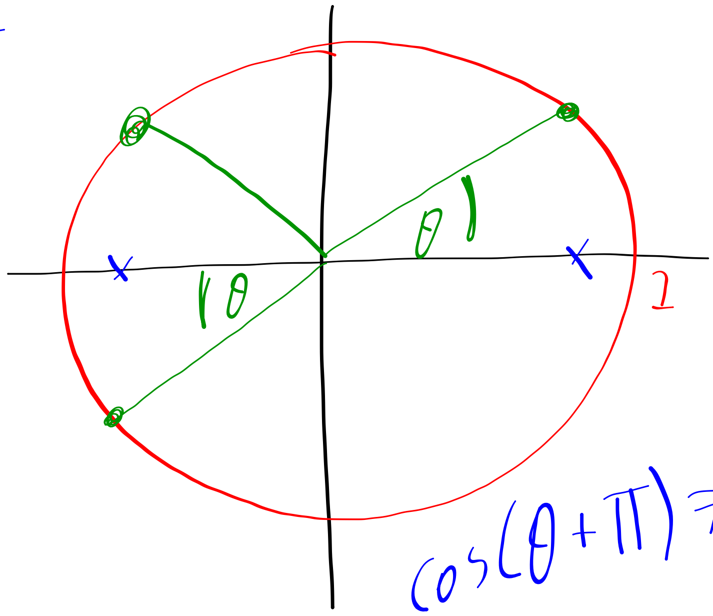
Consolidation

Are They Equivalent?

Does $\cos(\theta + \pi) = \cos\theta$?

***Use the unit circle.**

When using unit circle $x = \cos\theta$



$\cos(\theta + \pi) \neq \cos\theta$

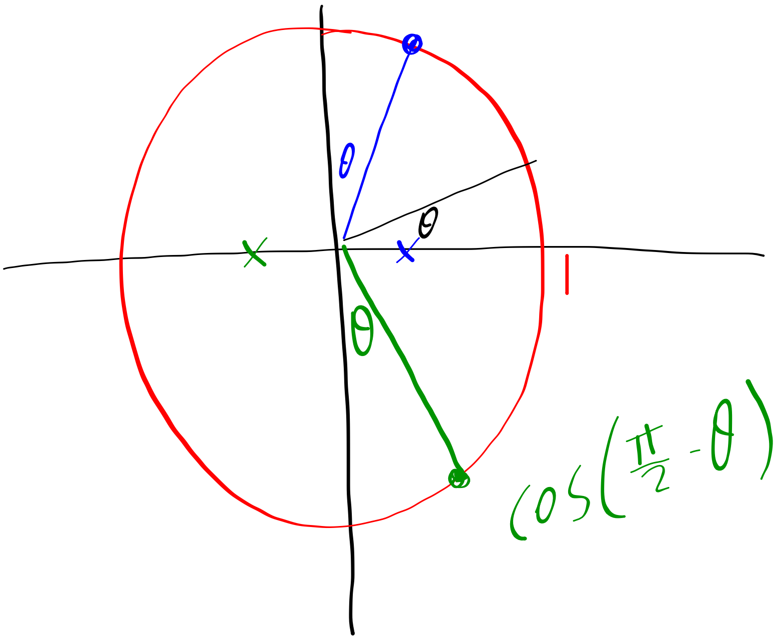
$\cos(\theta + \pi) = \cos(\pi - \theta)$ ✓

Consolidation

Are They Equivalent?

Does $\cos\left(\frac{\pi}{2}-\theta\right) = -\cos\left(\theta - \frac{\pi}{2}\right)$?

***Use the unit circle.**



$\cos\left(\frac{\pi}{2}-\theta\right) \neq -\cos\left(\theta-\frac{\pi}{2}\right)$

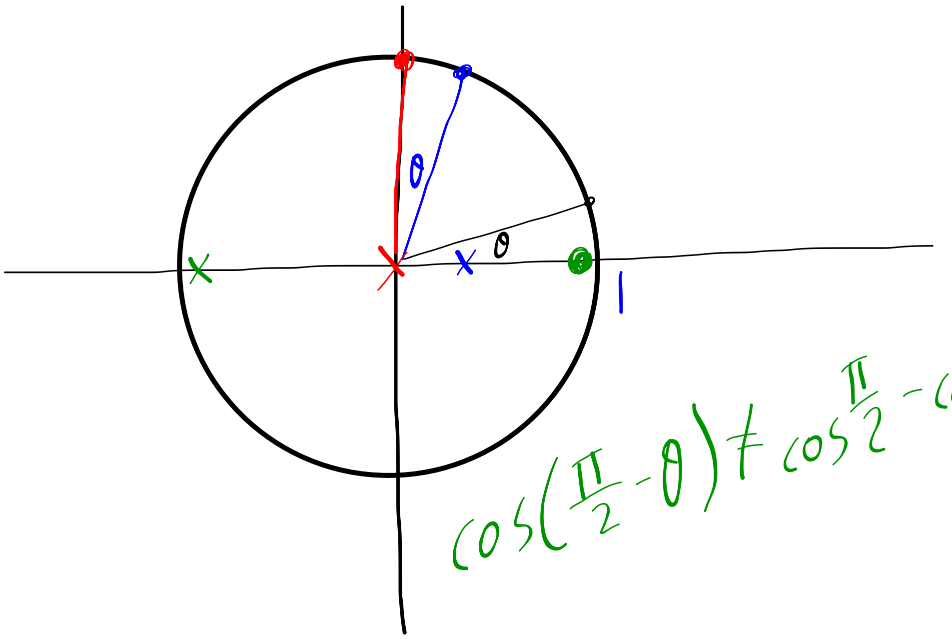
Consolidation

Are They Equivalent?

Does $\boxed{\cos\left(\frac{\pi}{2}-\theta\right)} = \underbrace{\cos\frac{\pi}{2}}_{0} - \cos\theta$?

$0 -$

***Use the unit circle.**



$\cos\left(\frac{\pi}{2}-\theta\right) \neq \cos\frac{\pi}{2} - \cos\theta$

Consolidation

Practice Questions

Pg. 392

1 - 3, 5, 7