Learning Goal: I will calculate and interpret average and instantaneous rates of change of sinusoidal functions.

Minds On: Interpret the graph qualitatively

Action: Note and examples

Consolidation: Exit Question

Minds On

Look at the graph for example 1.

Discuss with your partner what is happening at various stages of the graph.

You do not have to do any calculations, but you should discuss relative speeds.

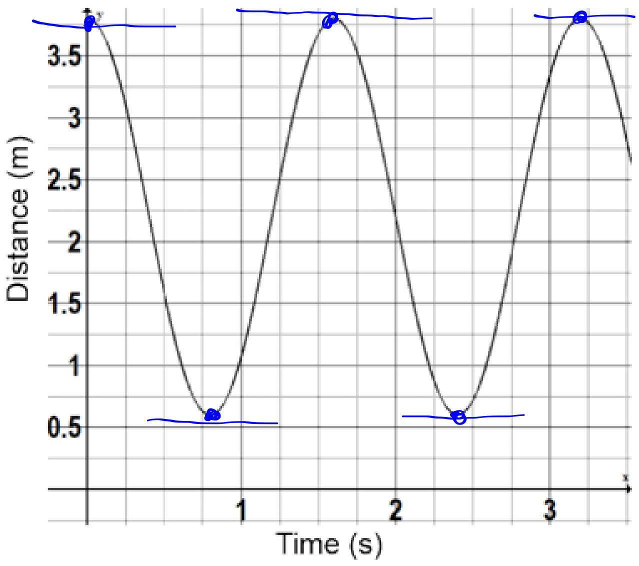
Action

Rates of Change in Trigonometric Functions

Example 1: Melissa used a motion detector to measure the horizontal distance between her and child on a swing. She stood in front of the child and recorded the distance, d(t), in metres over a period of time, t, in seconds. The data she collected are given in the following tables and are shown on the graph below. Use the data and the graph to discuss how the speed of the child changed as the child swung back and forth.

Time(s)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
Distance (m)	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6	0.72	1.07	1.59

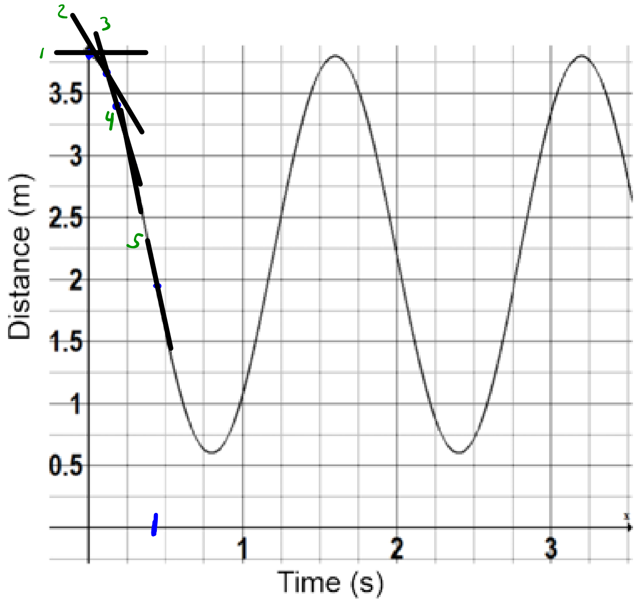
Time(s)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
Distance (m)	2.2	2.81	3.33	3.68	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.71	0.6



Farthest and Closest Points

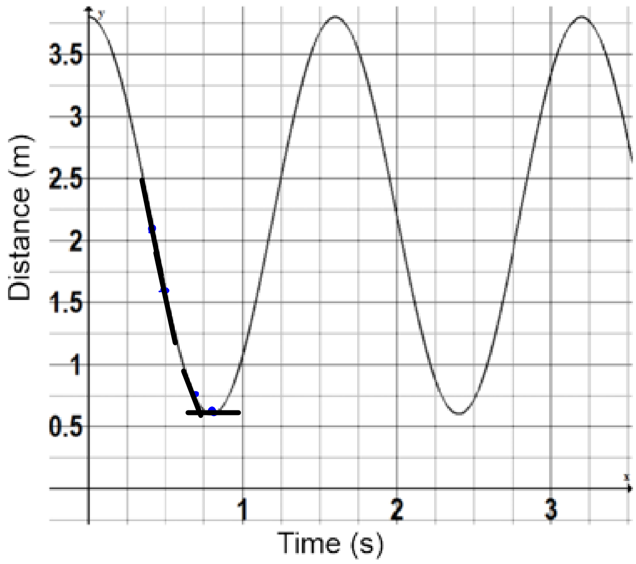
She is stopped @ each of these points.

roc = 0



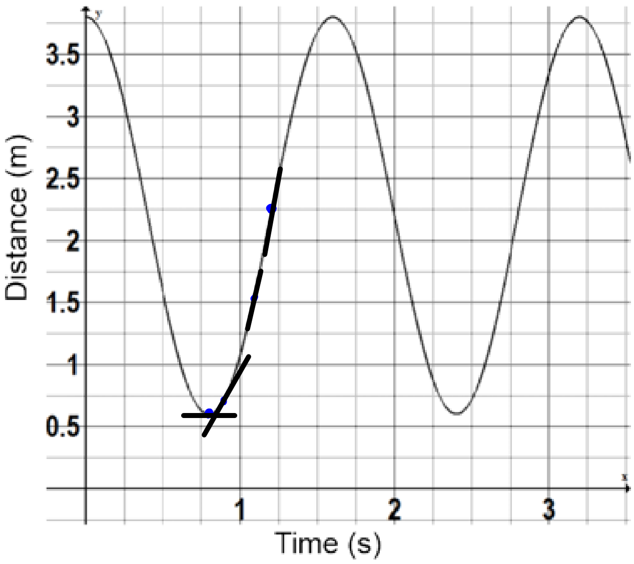
0 s to 0.4 s

Her speed is increasing and she is moving towards her mother at an increasing rate.



0.4 s to 0.8 s

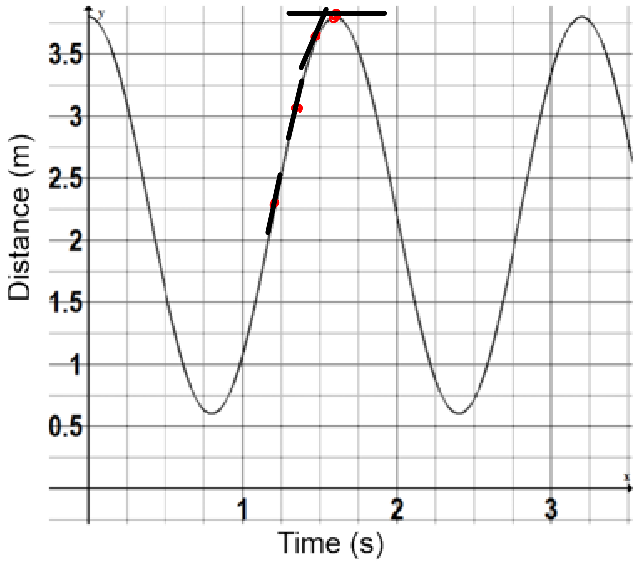
She is slowing down.
She is still moving towards her mother, now at a decreasing rate.



0.8 s to 1.2 s

Her speed is increasing.

She is moving away from her mother at an increasing rate.

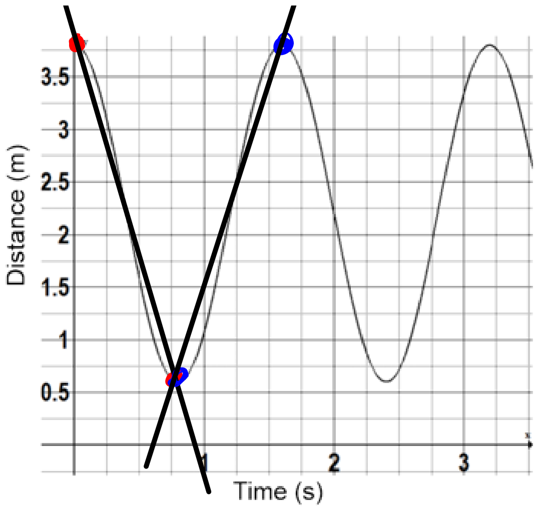


1.2 s to 1.~~8~~6 s

Her speed is decreasing.

She is moving away from her mother at a decreasing rate.

Example 2: Calculate the child's average speed over the intervals of time as the child swung toward and away from the motion detector on the first swing.



Toward

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$=\frac{0.6-3.8}{0.8-0}$$

$$=-4\ m/s$$

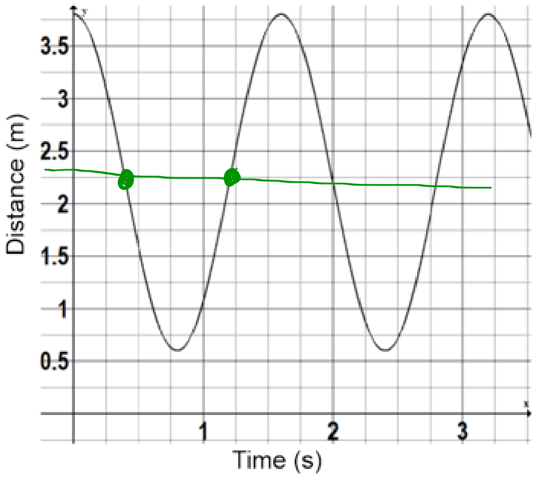
Away

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$=\frac{3.8-0.6}{1.6-0.8}$$

$$=4\ m/s$$

Example 3: To model the motion of the child on the swing, Melissa determined that she could use the equation $d(t) = 1.6\cos\left(\frac{\pi}{0.8}t\right) + 2.2$, where d(t) is the distance from the child to the motion detector, in metres, and t is the time, in seconds. Use this equation to estimate when the child was moving the fastest and what speed the child was moving at this time.



$d(t) = 1.6\cos\left(\frac{\pi}{0.8}t\right) + 2.2$

fastest time: along the axis
between a max + a min

The rate of change of a sinusoidal function is always highest halfway between the maximum and minimum values.

This is also each point along the axis of the function.

Consolidation

For the function below, determine a point when the instantaneous rate of change is

a) positive between $x=0$ and 2

b) negative between $x=2$ and $x=4$

$$f(x) = -2\cos\left(\frac{\pi x}{2}\right) + 4$$

c) zero when $x=0, 2, 4$

$f(x) = -2\cos\left(\frac{\pi}{2}x\right) + 4$

d) greatest

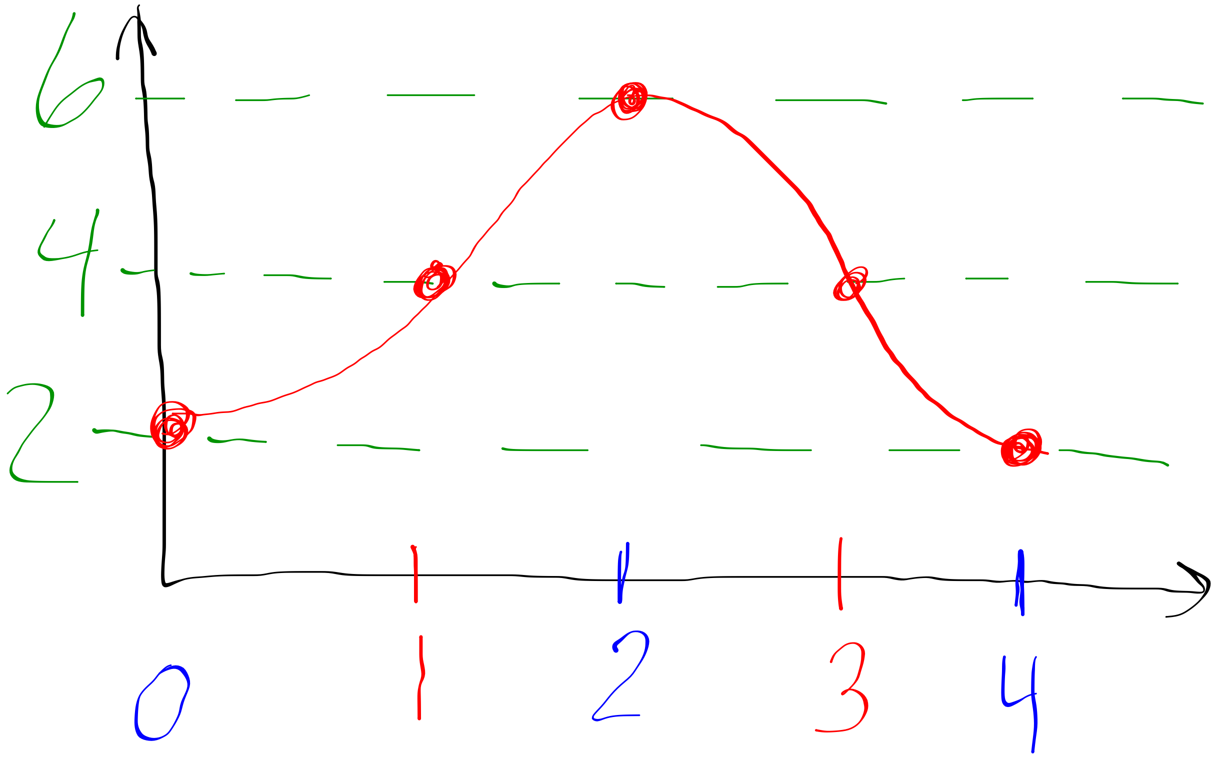
when $x=1$ and 3

axis $= 4$

max $= 6$

min $= 2$

period $= \frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2}} = 2\cancel{\pi} \times \frac{2}{\cancel{\pi}} = 4$


6
4
2
0
1
2
3
4

Pg. 369

1, 2, 5, 6, 8, 12