Learning Goal: I will be able to use transformations to sketch the graphs of the primary trig functions in radians. Given a data set, I will be able to model the data using a sinusoidal function.

Minds On: Activate - a, k, c, d...what do they do, what order do we do them in - look at graph!

Action: Note - transformations of functions

Consolidation: Exit Question

RAFT

Please read or work quietly on something until 10:30.

Minds On

$$y = a \sin(k(x - d)) + c$$

What effect do the parameters a, k, cand d have on a graph?

a → +/− reflects on x-axis
→ vertical stretches + compressions
* → amplitude
k → +/− reflects on y-axis
→ horizontal stretches + compressions
* → period = $\frac{2\pi}{k}$
d → shifts curve left + right
c → shifts curve up and down
* → equation of the axis

Which coordinate (x or y) does each parameter apply to?

a → y
k → x
d → x
c → y

What operation does each parameter impose?

a → multiply by y
k → divide by x
d → add to x
c → add to y

Action

Example 1: Sketch these transformations that are applied to the graph of y = sin x, where 0 ≤ x ≤ 2π.

- A vertical stretch by a factor of 3
- A horizontal compression by a factor of ½ $k=2$
- A horizontal translation $\frac{\pi}{6}$ to the left
- A vertical translation 1 down

$$y = 3\sin\left(2\left(x+\frac{\pi}{6}\right)\right)-1$$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\rightarrow$

$\frac{x}{2}-\frac{\pi}{6}$	$3y-1$
$-\frac{\pi}{6}$	-1
$\frac{\pi}{12}$	2
$\frac{\pi}{3}$	-1
$\frac{7\pi}{12}$	-4
$\frac{5\pi}{6}$	-1

$\frac{\pi}{2}$

$$\frac{\frac{\pi}{2}}{2} - \frac{\pi}{6}$$
$$= \frac{\pi}{4} - \frac{\pi}{6}$$
$$= \frac{3\pi}{12} - \frac{2\pi}{12}$$
$$= \frac{\pi}{12}$$

π

$$\frac{\pi}{2} - \frac{\pi}{6}$$
$$= \frac{3\pi}{6} - \frac{\pi}{6}$$
$$= \frac{2\pi}{6}$$
$$= \frac{\pi}{3}$$

$\frac{3\pi}{2}$

$$\frac{\frac{3\pi}{2}}{2} - \frac{\pi}{6}$$
$$= \frac{3\pi}{4} - \frac{\pi}{6}$$
$$= \frac{9\pi}{12} - \frac{2\pi}{12}$$
$$= \frac{7\pi}{12}$$

2π

$$\frac{2\pi}{2} - \frac{\pi}{6}$$
$$= \pi - \frac{\pi}{6}$$
$$= \frac{6\pi}{6} - \frac{\pi}{6}$$
$$= \frac{5\pi}{6}$$

Action

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$y = 3\sin\left(2\left(x + \frac{\pi}{6}\right)\right) - 1$

Solution A: Apply transformations to key points of the parent function

axis: -1

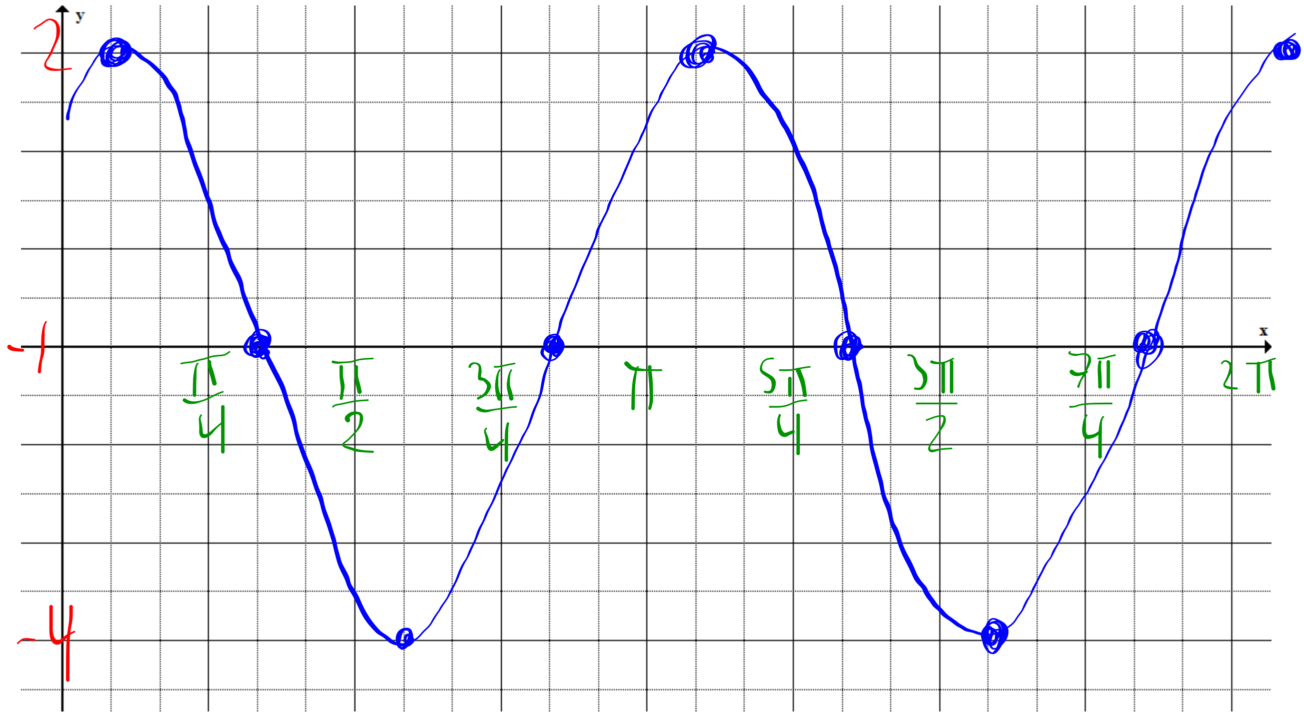
amplitud: 3

	$\frac{x}{2} - \frac{\pi}{6}$	$3y - 1$
	$-\frac{\pi}{6}$	-1
15°	$\frac{\pi}{12}$	2
60°	$\frac{\pi}{3}$	-1
105°	$\frac{7\pi}{12}$	-4
150°	$\frac{5\pi}{6}$	-1

period $= \frac{2\pi}{k}$

period $= \frac{2\pi}{2}$

period $= \pi$



over 3 each time

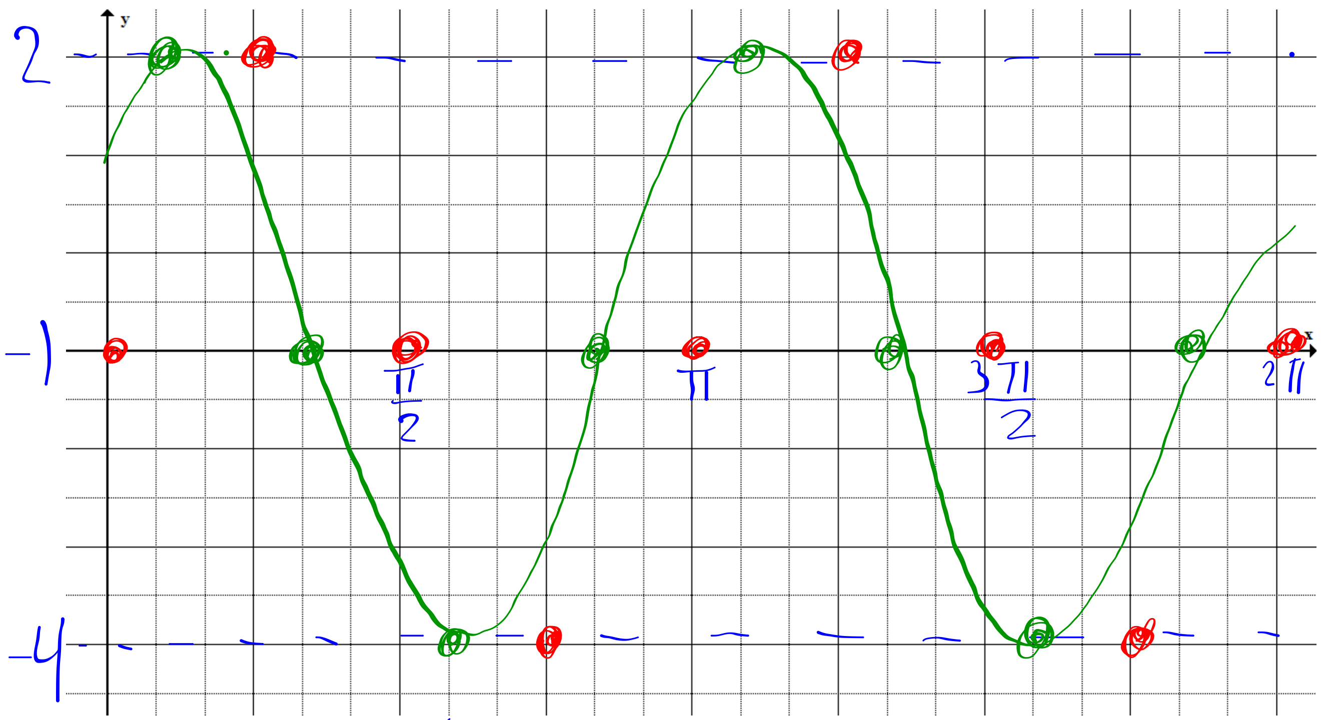
Action

Example 1: Sketch these transformations that are applied to the graph of y = sin x, where $0 \leq x \leq 2\pi$.

- A vertical stretch by a factor of 3
- A horizontal compression by a factor of ½
- A horizontal translation $\frac{\pi}{6}$ to the left
- A vertical translation 1 down

sketch w/ everything except d, then shift points

Solution B: Use the features of the transformed function



period $= \frac{2\pi}{2} = \pi$

axis $= -1$

amplitude $= 3$

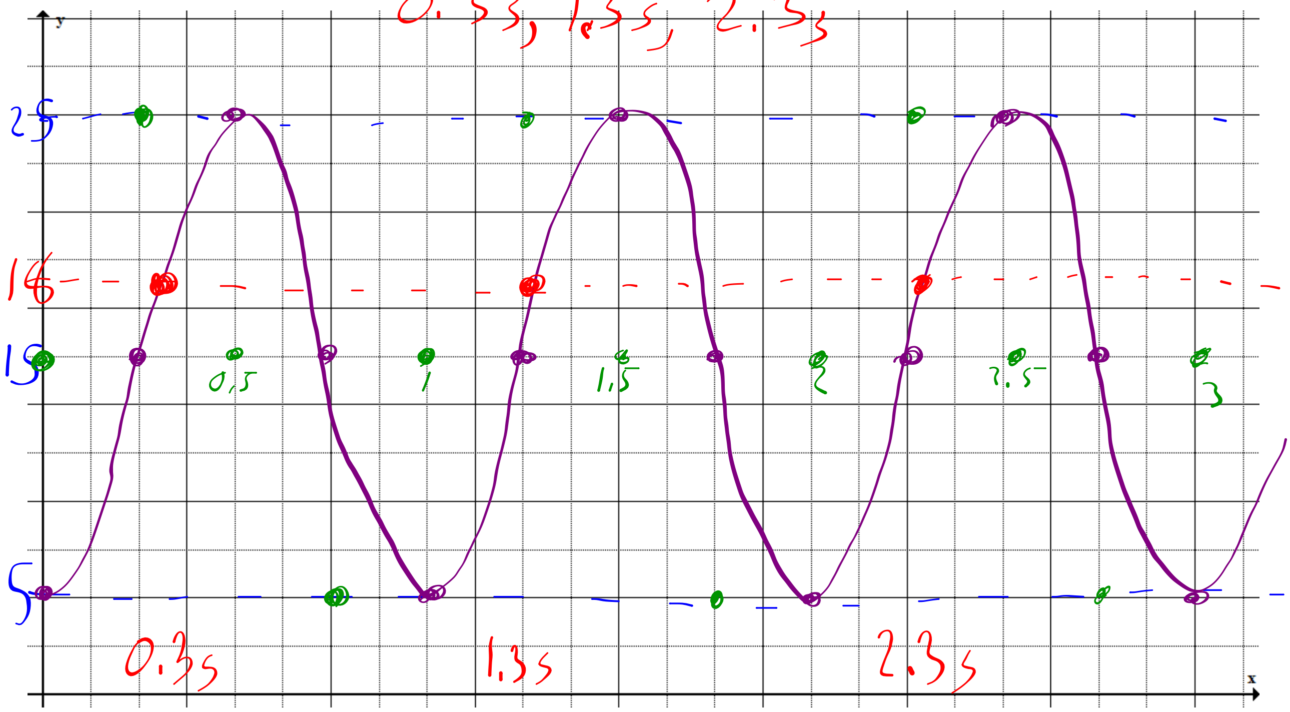
plot points ignoring the horizontal shift

now shift each point $\frac{\pi}{6}$ left

Action

Example 2: A mass on a spring is pulled toward the floor and released, causing it to move up and down. Its height, in centimetres, above the floor after t seconds is given by the function $h(t) = 10\sin(2\pi t + 1.5\pi) + 15, where\ 0 \le t \le 3$. Sketch a graph of height versus time. Then use your graph to predict when the mass will be 18 cm above the floor as it travels in an upright direction.

0.3s, 1.3s, 2.3s



$h(t) = 10\sin(2\pi(t + 0.75)) + 15$

period $= \frac{2\pi}{2\pi} = 1$

shift points left by 0.75 OR right by 0.25

axis = 15

amplitude = 10

Action

Example 3: The following graph shows the temperature in Nellie's dorm room over a 24 h period.

Determine the equation of this sinusoidal function.

axis $= \frac{25+13}{2} = 19$

amplitude $= 6$

period = 5-17 is half a period

period $= 24$

period $= \frac{2\pi}{k}$

$k = \frac{2\pi}{\text{period}}$

$k = \frac{2\pi}{24} = \frac{\pi}{12}$

$$\boxed{y = -6\cos\left(\frac{\pi}{12}(x-5)\right) + 19}$$

OR

$$\boxed{y = 6\sin\left(\frac{\pi}{12}(x-11)\right) + 19}$$

$$\boxed{y = 6\cos\left(\frac{\pi}{12}(x-17)\right) + 19}$$

Action

Summary of Key Ideas

- The graphs of functions of the form f(x) = a sin(k(x – d)) + c and f(x) = a cos(k(x – d)) + c are transformations of the parent functions y = sin(x) and y = cos(x), respectively.
- The parameters a, k, d, and c give useful information about transformations and characteristics of the function.

Transformations of the Parent Function	Characteristics of the Transformed Function
$\lvert a \rvert$ gives the vertical stretch/compression factor. If a < 0, there is also a reflection in the x-axis.	$\lvert a \rvert$ gives the amplitude.
$\left\lvert \frac{1}{k} \right\rvert$ gives the horizontal stretch/compression factor. If k < 0, there is also a reflection in the y-axis.	$\left\lvert \frac{2\pi}{k} \right\rvert$ gives the period.
d gives the horizontal translation.	d gives the horizontal translation.
c gives the vertical translation.	y = c gives the equation of the axis.

- If the independent variable (x, t, etc) has a coefficient other than +1, the argument (angle) must be factored to separate the values of k and d. For example,

y = 3 cos(2x + π) should be changed to $y = 3\cos\left(2\left(x + \frac{\pi}{2}\right)\right)$.

Consolidation

Pg. 343

1, 4, 6, 8