What's Going On?

Checking In

Minds on Yesterday's Final Question

Action! Finding Patterns

Consolidation The Fibonacci Sequence

Learning Goal - I will be able to recognize patterns in sequences that are not strictly arithmetic or geometric.

Minds on

Yesterday's Final Question

multiplication!!

The 5th term of a geometric sequence is 45 and the 8th term is 360. Determine the 20th term.

$t_5 = 45$

$t_8 = 360$

$\frac{360}{45} = 8$

$8 - 5 = 3$ (terms)

$\sqrt[3]{8} = 2$ (ratio)

Find general term

$t_n = ar^{n-1}$

$45 = a \times 2^4$

$\frac{45}{16} = \frac{16a}{16}$

$a = 2.8125$

To get a_1 divide 45 by 2, 5 times or... use formula.

$$\therefore t_n = 2.8125 \times 2^{n-1}$$

$$t_{20} = 2.8125 \times 2^{19}$$

$$t_{20} = 1,474,560$$

Action!

Finding Patterns

Not all patterns are arithmetic or geometric.

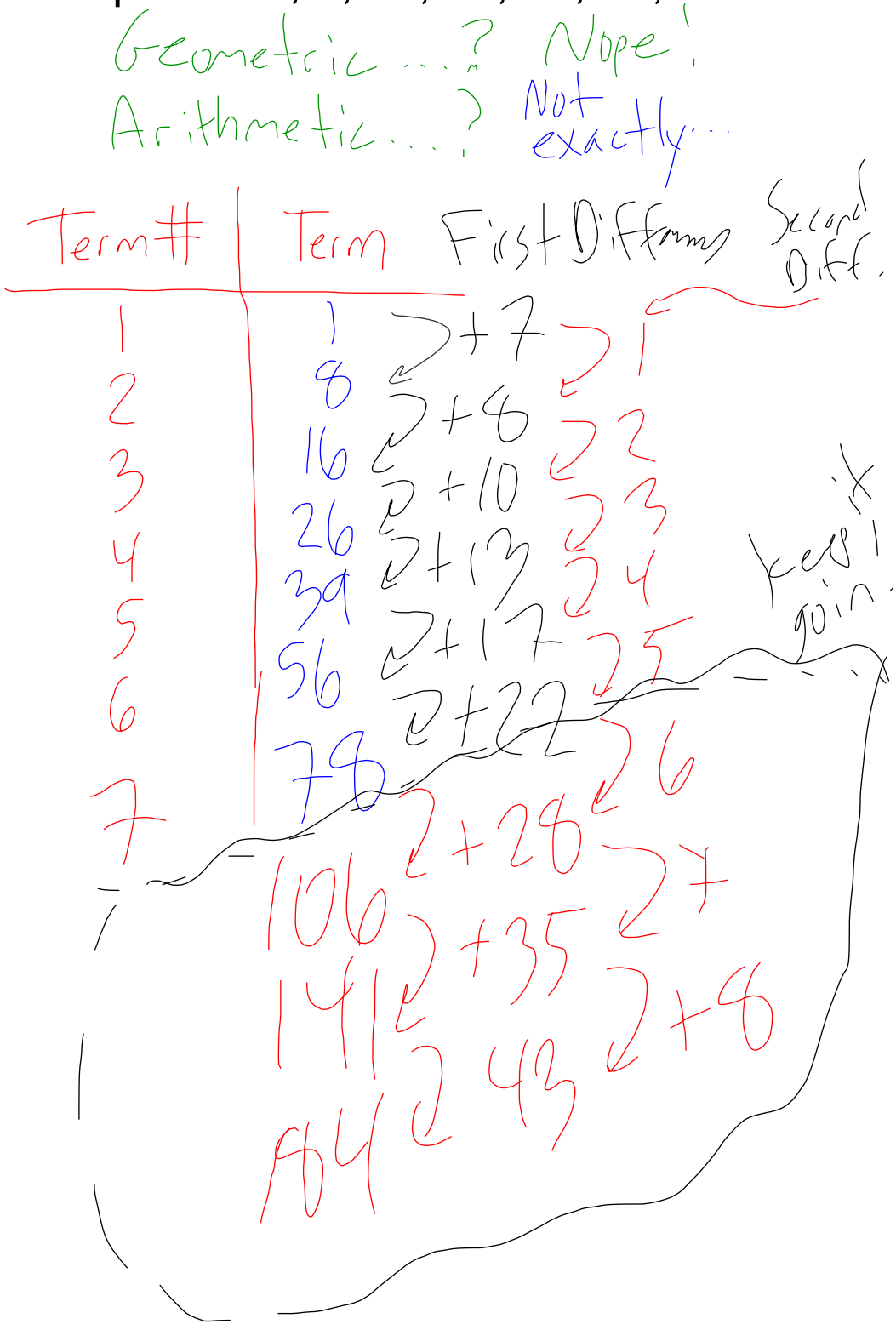
Some are a bit of a combination of both, some follow their own set of rules all together (see Fibonacci's sequence)

We can't always find the general term, but we can often create a recursive formula.

Action!

Finding Patterns

Determine the **next three terms** in the sequence 1, 8, 16, 26, 39, 56, 78 ...



Action!

Finding Patterns

Determine the **recursive formula** of the sequence

5, 14, 41, 122, 365, 1094, 3281 ...

$t_1 = a, t_n = ? \Rightarrow t_1 = 5, t_n = ?$

arithmetic? NOPE!

geometric?

Term #	Term	approx
1	5	×2.8
2	14	×2.9
3	41	×3.0
4	122	×3.0
5	365	×3.0
6	1094	×3.0
7	3281	

What would happen if we multiplied by 3?

Term #	Term	$t_{n-1} \times 3$
1	5	
2	14	15
3	41	42
4	122	123
5	365	366
6	1094	1095
7	3281	3282

So..... if we multiply t_{n-1} by 3, then subtract 1... we get t_n!

$\therefore t_n = 3t_{n-1} - 1$

Action!

Finding Patterns

Determine the **general term** of the sequence

$$\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}, \frac{13}{49}, \frac{15}{64} \ldots$$

When we have sequences where the terms are fractions, it is often helpful to consider the numerator and denominator separately!

Numerator (arithmetic)

$a = 3 \quad d = 2$

$t_n = 3 + (n-1)2$

$t_n = 3 + 2n - 2$

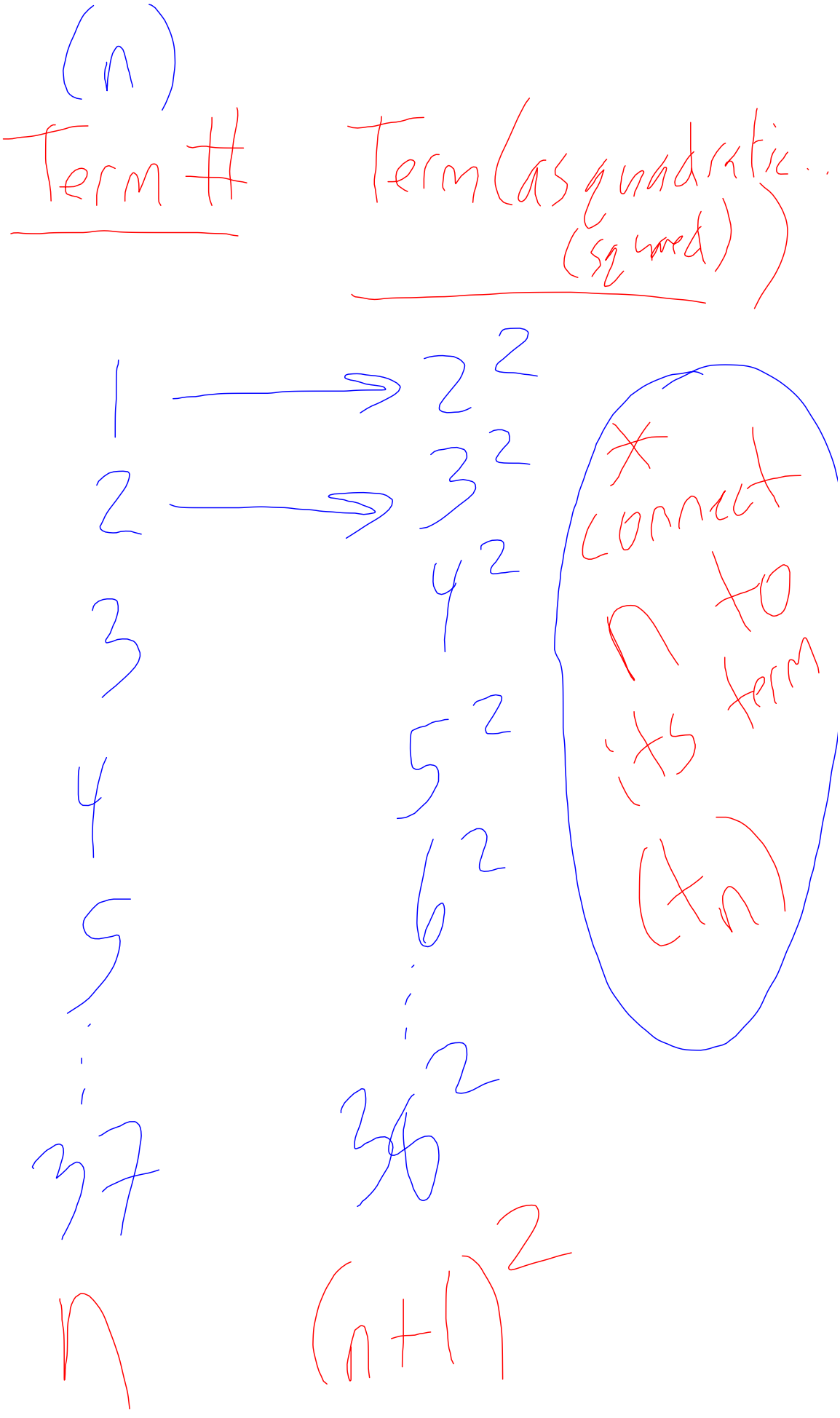
$t_n = 2n + 1$

Denominator

Term #	Term	First Diffs	Second Diffs
1	4	5	2
2	9	7	2
3	16	9	2
4	25	11	2
5	36	13	2
6	49	15	2
7	64		

Second diff. constant . . .

quadratic.


(n)
Term #
Term (as quadratic... (squared))
1 → 2²
2 → 3²
3
4
5
⋮
37
n
4²
5²
6²
⋮
36²
(n+1)²
* connect n to term (×5) (+n)

∴ denominator is $(n+1)^2$

$$t_n = \frac{2n+1}{(n+1)^2}$$

$n \geq 1$

Consolidation

The Fibonacci Sequence

The Fibonacci Sequence is the series of numbers:

0, 1, 1, 2, 3, 5, 8 ...

a. Determine the next 3 terms in the Fibonacci Sequence.

13, 21, 34, 55, 89

b. Determine the recursive formula for the Fibonacci Sequence.

$t_1 = 0, t_2 = 1, t_n = t_{n-1} + t_{n-2}$

$n > 2$

The "next" term is the previous two terms added together! BUT!!!!!! we can't start that as the recursive formula UNTIL the third term. Otherwise, we don't have two previous terms :)