Future Value Annuities

Present Value Annuity

A series of withdrawals from an investment, or a series of payments on a loan, made at regular intervals.

Formula

$$PV = R \times \left(\frac{1 - (1 + i)^{-n}}{i} \right)$$

PV represents the present value of the annuity

R represents the regular payment / withdrawal made at each compounding period

i is the interest rate per compounding period, as a decimal

n is the total number of compounding periods

Example

How much would you need to invest at 8.3% compounded annually to provide $500 per year for the next 10 years?

$R = 500$

$i = 0.083$

$n = 10$

$$PV = 500 \times \left(\frac{1 - (1 + 0.083)^{-10}}{0.083} \right)$$

Rounded

$$PV = 500 \times \left(\frac{1 - 0.45505}{0.083} \right)$$

$PV = 3310.24$ [3310.11]

Example

How much would you need to invest at 5.4% interest, compounded quarterly, to provide $1,000 every 3 months for the next 4 years?

$R = 1000$

$i = \frac{0.054}{4}$

$n = 4 \times 4 = 16$

$PV = 1000 \times \left(\frac{1 - (1 + \frac{0.054}{4})^{-16}}{\frac{0.054}{4}} \right)$

$* \{ PV = 1000 \times \left(\frac{1 - (1.0135)^{-16}}{0.0135} \right)$

$PV = 14303.67$

Example

You borrow $200,000 from the bank to purchase yacht, times are good. The bank charges 6.6% compounded monthly and you will take 20 years to pay off the loan.

a. How much will each monthly payment be?

$PV = 200000$

$i = \frac{0.066}{12}$

$n = 12 \times 20$

$200000 = R \times \left(\frac{1 - (1 + \frac{0.066}{12})^{-240}}{\frac{0.066}{12}} \right)$

$200000 = R \times \left(\frac{1 - (1.0055)^{-240}}{0.0055} \right)$

$R = 1502.94$

b. How much interest will you have paid, in total, over the term of the loan?

We pay 1502.94 every month for 20 years (240 times!)

We pay $1502.94 \times 240 = 360705.60$

Total interest is $360705.60 - 200000 = 160705.60$