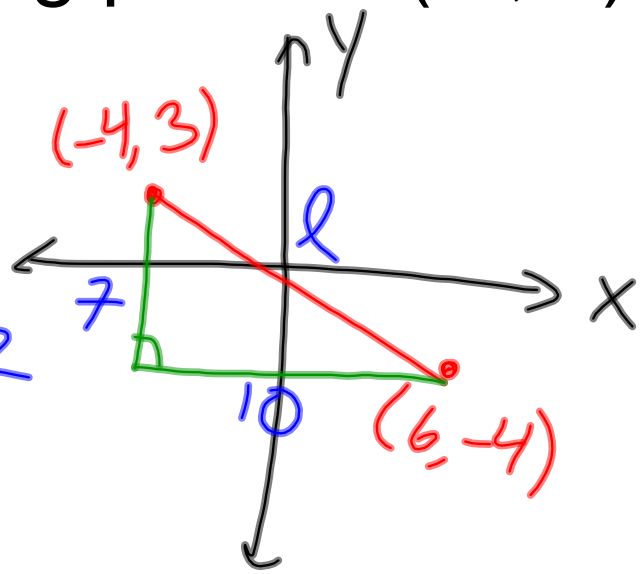
Analytic Geometry

Find the length of the line segment joining points A(-4, 3) and B(6, -4).



$\ell = \sqrt{7^2 + 10^2}$

$\ell^2 = 7^2 + 10^2$

$\sqrt{\ell^2} = \sqrt{149}$

$\ell \doteq 12.2$

$$\ell = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\ell = \sqrt{(-7)^2 + (10)^2}$$

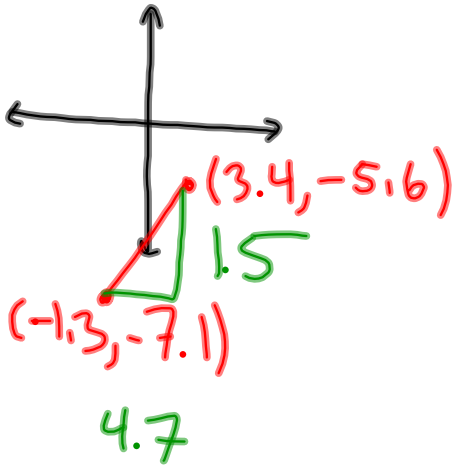
$$\ell = \sqrt{49 + 100}$$

$$\ell = \sqrt{149}$$

$$\ell \doteq 12.2$$

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A line segment goes through the points (3.4, -5.6) and (-1.3, -7.1). What's the length of the segment to two decimal places?



$l = \sqrt{(-1.3-3.4)^2 + (-5.6-(-7.1))^2}$

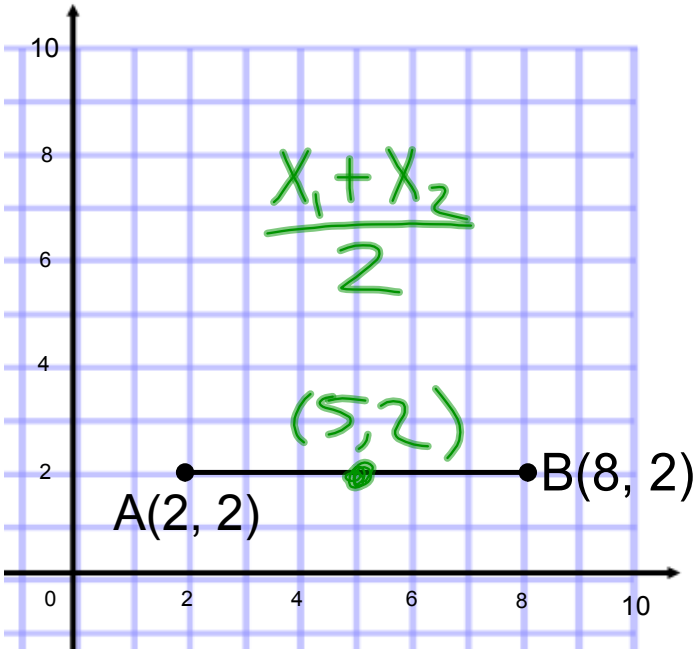
$l = \sqrt{(-4.7)^2 + (1.5)^2}$

$l = \sqrt{22.09 + 2.25}$

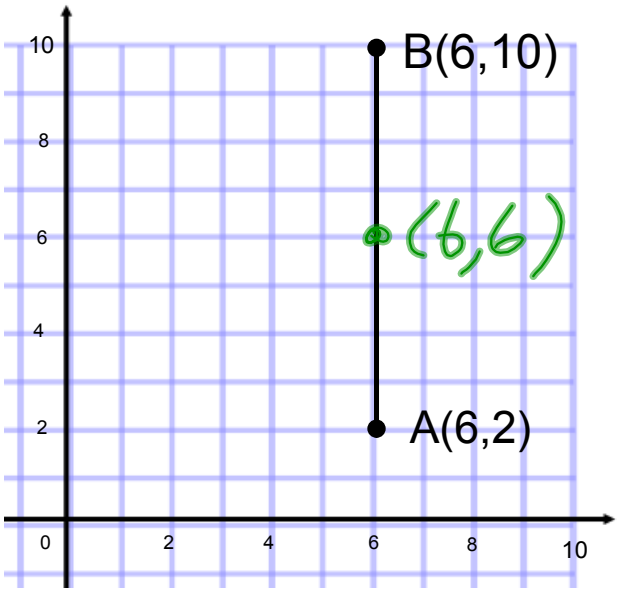
$l = \sqrt{24.34}$

$l \approx 4.93$

What's the Midpoint?



What's the Midpoint?



Find the midpoint of PQ, where P(1, 7) and Q(5, 9)...

$$M_{PQ} = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2}\right)$$

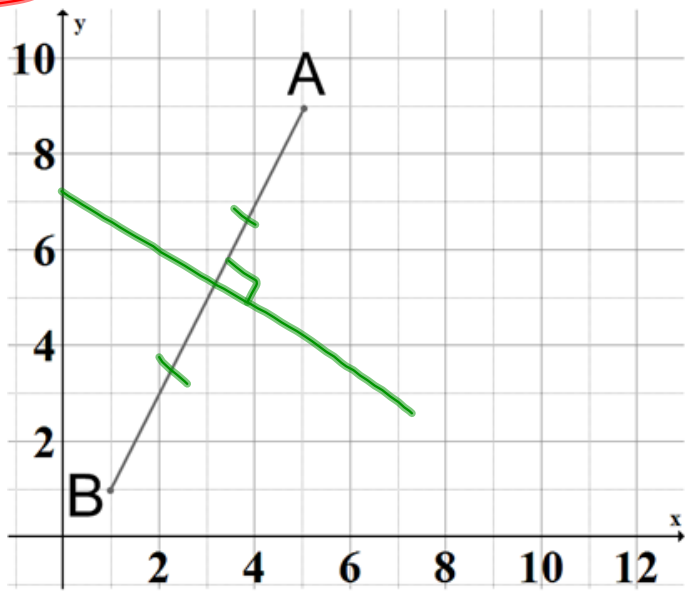
$$= \left(\frac{1+5}{2}, \frac{7+9}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{16}{2}\right)$$

$$= (3, 8)$$

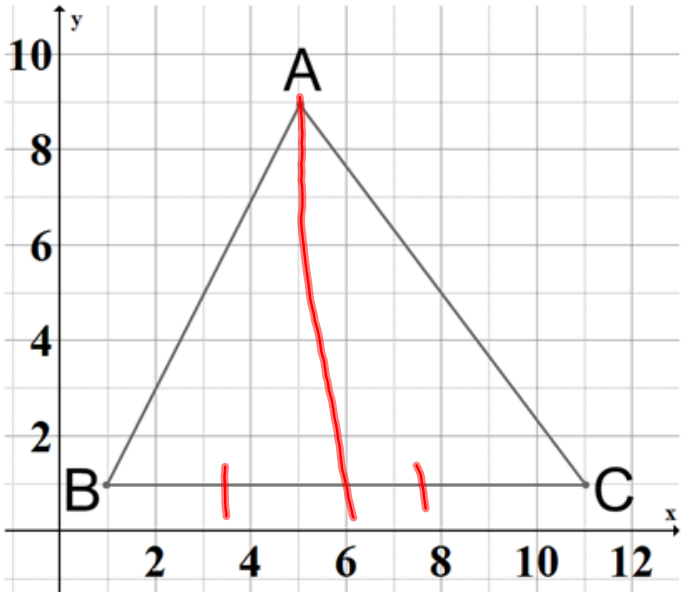
Right Bisector

a line that "cuts" another line in half at a 90^{o} angle (perpendicularly)



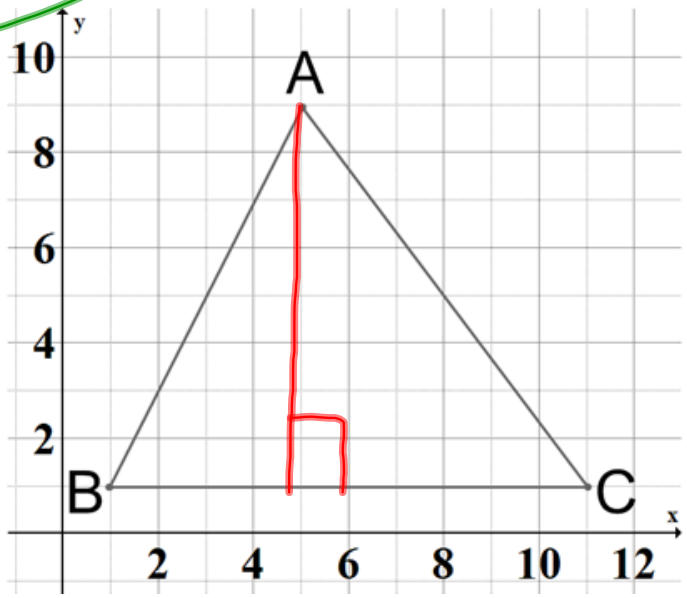
Median

a line dropped from a vertex of a triangle that lands on *the opposite* midpoint

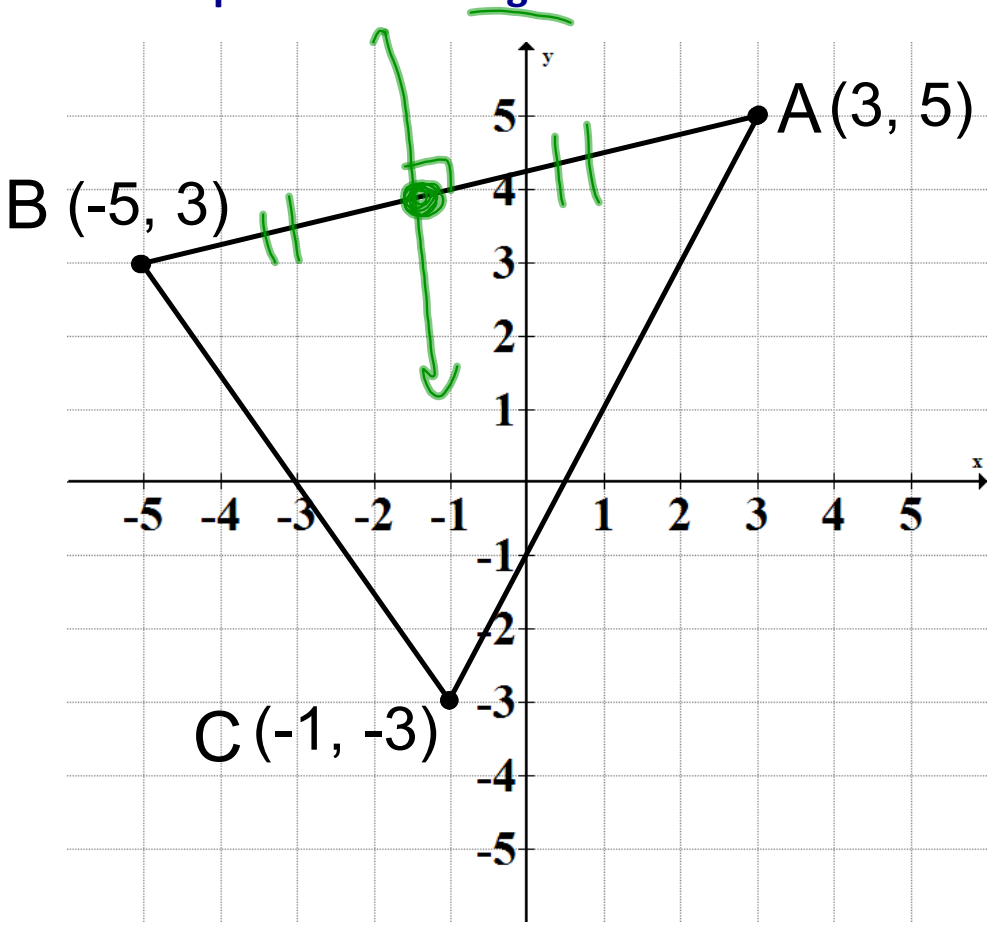


Altitude

The perpendicular height of a triangle



Determine the equation of the right bisector of side AB.



M_{AB} is on our line

$$M_{AB} = \left(\frac{-5+3}{2}, \frac{3+5}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{8}{2}\right)$$

$$= (-1, 4)$$

The slope of our line is the slope $\perp$ to m_{AB}

$$m_{AB} = \frac{5-3}{3-(-5)}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

$\perp m_{AB} = -4$

∴ our slope is -4 and a point is $(-1,4)$. Use point-slope equation.

$$y - y_1 = m(x - x_1)$$

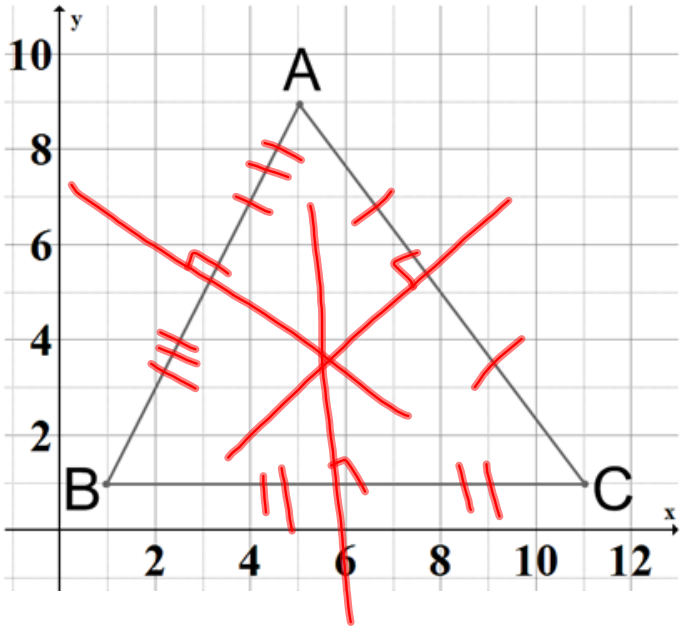
$$y - (4) = -4(x - (-1))$$

$$y - 4 = -4x - 4$$

$$4x + y = 0$$

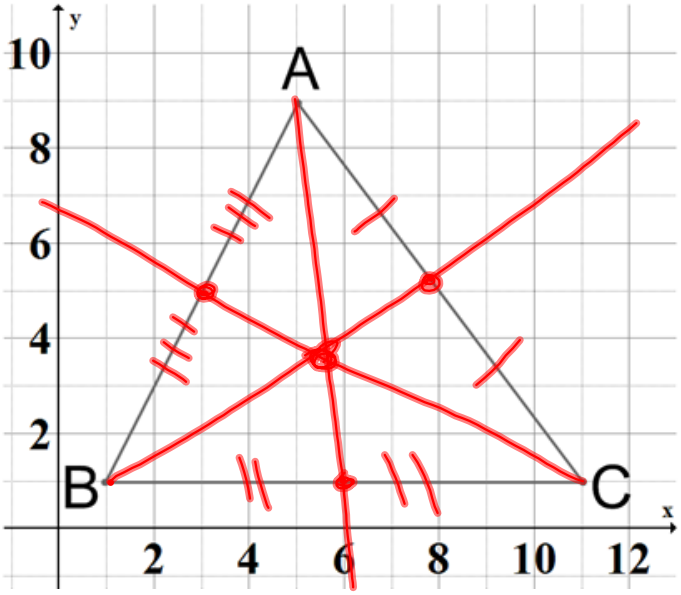
Circumcentre

The point of intersection of **the right bisectors** of a triangle



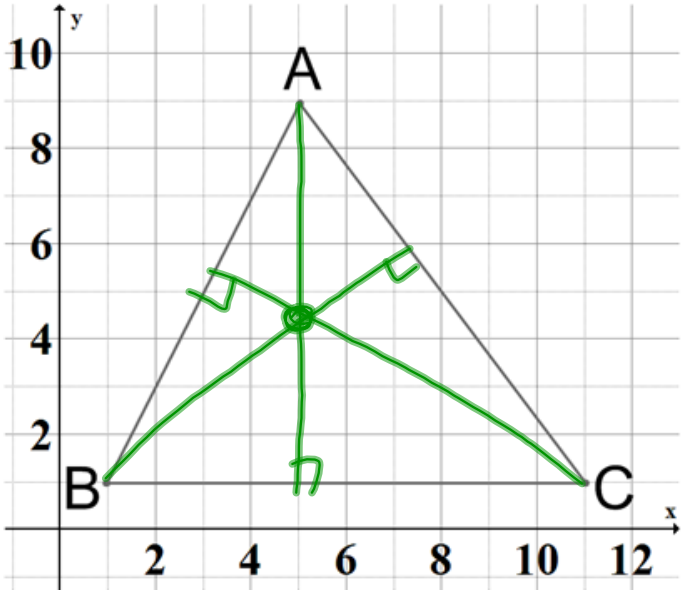
Centroid

The point of intersection of **the medians** of a triangle



Orthocentre

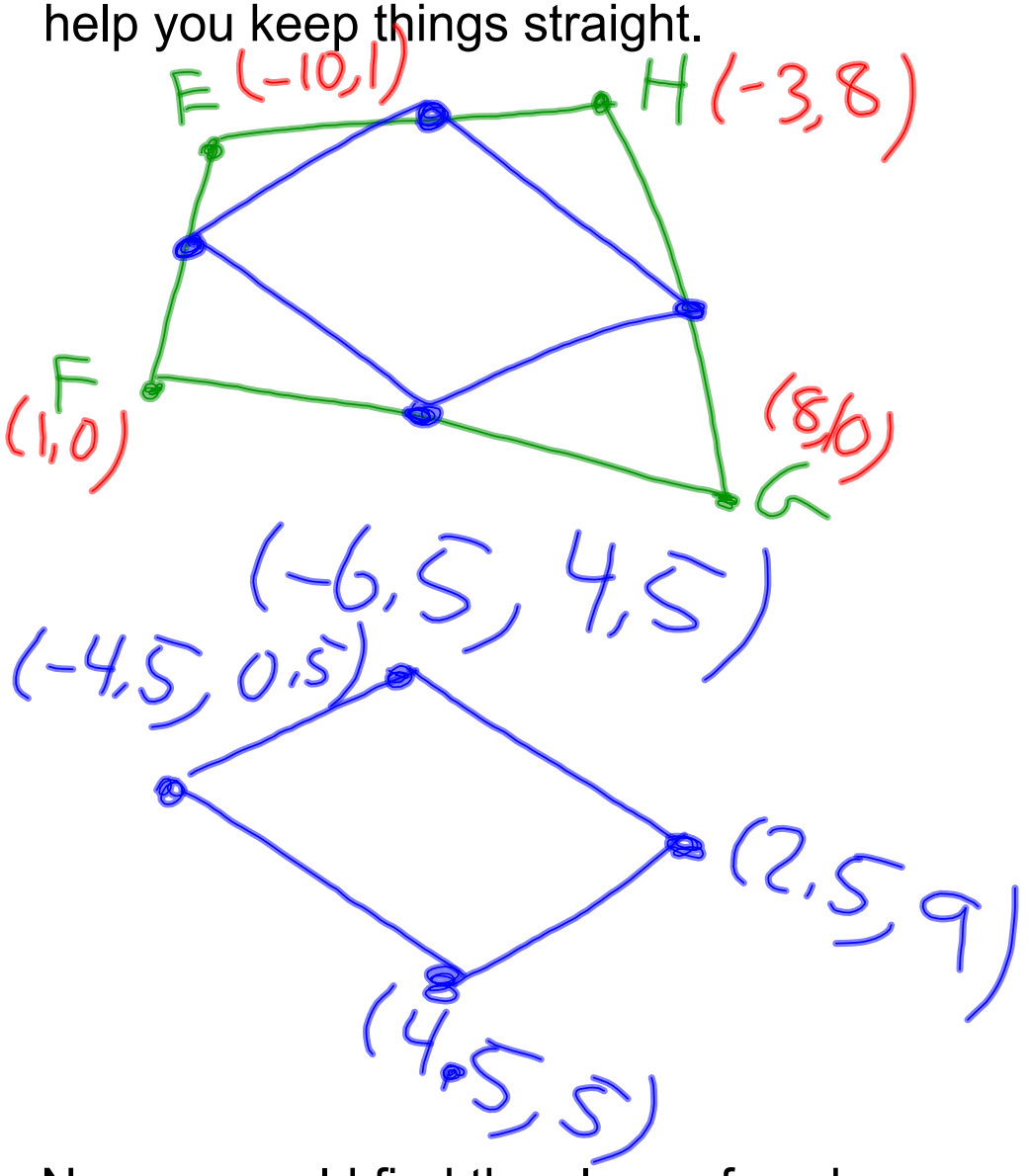
The point of intersection of **the altitudes** of a triangle



Quadrilateral EFGH has vertices at:
E(-10, 1), F(1, 0), G(8, 10) and H(-3, 8)

What shape is produced if the midpoints of adjacent sides are joined? **Explain how you know.**

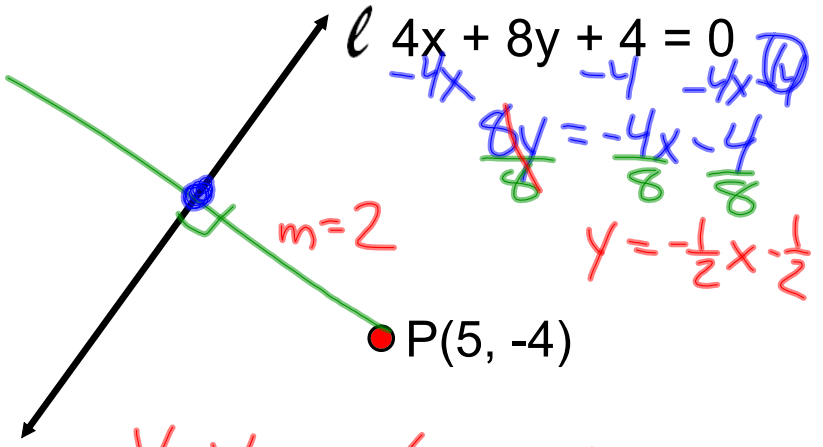
You may want to sketch a rough diagram to help you keep things straight.



Now we could find the slope of each segment and the length of each segment to determine what kind of quadrilateral we have.

Distance from a Point to a Line

Determine the shortest distance from the point to the line.



$-4x \quad -4 \quad -4x$

$\frac{8y}{8} = \frac{-4x}{8} - \frac{4}{8}$

$y = -\frac{1}{2}x - \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - (-4) = 2(x - 5)$

$y + 4 = 2x - 10$

$y = 2x - 14$

sub $y = 2x - 14$ into (1)

$4x + 8(2x - 14) + 4 = 0$

$4x + 16x - 112 + 4 = 0$

$20x = 108$

$x = \frac{27}{5} = 5.4$

$y = 2(5.4) - 14$

$y = -3.2$

Now we just have to find the distance from our original point to our new point using the length of a line segment formula.